

所別：地球物理研究所碩士班 一般生 科目：應用數學
學位在職生

1. Solve the following ordinary differential equations.

(1) $\cos y \frac{dy}{dx} = -\frac{x \sin y}{1+x^2}$ (10%)

(2) $xy' - y = x$ (10%)

(3) $y'' - y = 2e^x + 6e^{2x}$ (10%)

2. (10%) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}.$$

3. (10%) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$$

4. (10%) Find the directional derivative of f at P in the direction of \bar{a} , where

$$f = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, \quad P: (3, 0, 4), \quad \bar{a} = \hat{i} + \hat{j} + \hat{k}.$$

5. (10%) Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n} dA$,

where $\vec{F} = [x^2, 0, z^2]$, S the surface of the box $|x| \leq 1, |y| \leq 3, |z| \leq 2$.

6. (14%) Find the Fourier series of the function $f(x)$, which is assumed to have the period 2π , and $f(x) = x$ ($-\pi < x < \pi$).

7. (16%) Apply method of separating variables to solve the one - dimensional wave equation

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2},$$

with the two boundary conditions

$$u(0, t) = 0, \quad u(L, t) = 0 \quad \text{for all } t,$$

and the two initial conditions

$$u(x, 0) = f(x), \quad \left. \frac{\partial u(x, t)}{\partial t} \right|_{t=0} = g(x).$$