國立中央大學八十四學年度碩士班研究生入學試題卷

所別：大氣物理研究所 ______ 組 科目：應用數學 ______ 共 1 頁 第 1 頁

1. Solve the differential equation and satisfy the conditions,

$$x^2\frac{d^2y}{dx^2}-5x\frac{dy}{dx}+8y=0 \quad ;$$

$$y(1)=5 \quad ; \quad \frac{dy}{dx}(1)=18$$

(10%)

2. Using Laplace transforms, find x(t) and y(t) which satisfied the given equation and conditions,

$$\begin{cases}\dfrac{dx}{dt}+2\dfrac{dy}{dt}=t \\ \dfrac{d^2x}{dt^2}-y=e^{-t}\end{cases} \quad ;$$

$$x(0)=3 \quad , \frac{dx}{dt}(0)=-2 \quad ; y(0)=0$$

(15%)

3. Find the eigenvalues and eigenvectors of the following matrices,

$$\begin{bmatrix}32 & -24 & -8 \\ 16 & -11 & -4 \\ 72 & -57 & -18\end{bmatrix}$$

(15%)

4. Represent the following function f(x) by (1) Fouries sine series, (2) Fouries cosine series, and (3) complete Fouries series and sketches the responding periodic extension of f(x).

$$f(x)=x+1 \qquad 0\le x\le 1$$

(15%)

5. Use residue calculus to evaluate the integration

$$\int_{-\infty}^{\infty}\frac{e^{px}}{1+e^x}dx \qquad (0<p<1)$$

(15%)

6. Solve the partial differential equation with the conditions as following,

$$\frac{\partial^2T}{\partial x^2}=\frac{1}{\alpha^2}\frac{\partial T}{\partial t} \quad (t>0 \quad ; \quad x\ge 0)$$

Initial Contion : $T(x,0)=T_0$ (constant);

Boundary Condition : $T(0,t)=0$ $(t>0)$.

(15%)

7. Find the values of the following integration,

(a) $\oint_C[(1+y)z\,dx+(1+z)x\,dy+(1+x)y\,dz]$

where $C: x=\cos\theta, y=\sin\theta, z=1.$

(8%)

(b) $\oiint_S \bar{r}\bullet\hat{n}\,d\sigma$

where $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}; \quad S: x^2+y^2+z^2=4.$

(7%)