參考用

1. Determine the general solution of the ordinary differential equation (17%):

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2 .$$

2. By use of Laplace Transform, solve the following initial value problem (16%):

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=2, \quad y(0)=0, \frac{dy(0)}{dx}=1,$$

3. Use residue calculus to evaluate the following integral (20%):

$$\int_0^\infty \frac{x^{m-1}}{x+1}dx, \quad (0<m<1).$$

4. If $x=r\cos\theta$ and $y=r\sin\theta$, evaluate the following partial differential problems and express the results in terms of r and θ (20%):

(a) $\left(\frac{\partial\theta}{\partial x}\right)_y$　　(b) $\left(\frac{\partial r}{\partial y}\right)_x$

(c) $\left(\frac{\partial\theta}{\partial r}\right)_y$　　(d) $\left(\frac{\partial\theta}{\partial r}\right)_x$

5. Evaluate the Fourier series of the following functions with given interval (12%):

(a) $f(x)=e^x$ in the interval $(-\pi,\pi)$.
(b) $f(x)=e^x$ in the interval $(0,2\pi)$.

6. If $\vec{A}=\frac{1}{r^3}\vec{r}$ (where $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$), and $I=\oiint_S \vec{A}\bullet d\vec{s}$, where S is a closed surface (15%):

(a) Determine the value of I if the origin point is located inside the closed surface S,
(b) Determine the value of I if the origin point is located outside the closed surface S.