

所別：水文科學研究所碩士班 一般生 科目：應用數學

1. Find general solution of

(1) $\int_0^\pi \int_0^{1-\cos\theta} r dr d\theta$ (5%)

(2) $\int_0^1 \int_{x^2}^x (1-xy) dy dx$ (5%)

(3) $y'' - 4y' + 4y = 0, y(0) = 3, y'(0) = 1$ (5%)

(4) $x^2 y'' - 1.5xy' - 1.5y = 0$ (5%)

(5) $y'' + y = 0, y(0) = 3, y(\pi) = -3$ (5%)

(6) $(2x - 4y + 5)y' + x - 2y + 3 = 0$ (5%)

2. (1) Find divergence of $e^{-xy}\vec{i} + e^{-yz}\vec{j} + e^{-zx}\vec{k}$ (5%)

(2) Find curl of $v = x\vec{i} + yz\vec{j} - (x^2 + z^2)\vec{k}$ (5%)

(3) Find gradient of $(x^2 + y^2 + z^2)^{-1/2}$ (5%)

(4) Find the first order partial derivatives of $u = e^x \cos y + xy$ (5%)

3. Solve

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 10x_2 - 3x_3 = 5$$

$$-x_1 + x_2 + x_3 = -3 \quad (10\%)$$

4. Let $u(x, y) = x^2 + y^2$, where $x = r \cos \theta$ and $y = r \sin \theta$. Find the partial derivatives of u with respect to r and θ up to the second order. (10%)

5. Find the solution of the equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, (-\infty < x < \infty, t > 0)$$

satisfying the initial conditions $u(x, 0) = \sin x$ and $\frac{\partial u(x, 0)}{\partial t} = 0$. (15%)

6. Find the solution of the initial-boundary value problem

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, (0 < x < \pi, t > 0)$$

$$u(x, 0) = \sin x, (0 \leq x \leq \pi) \quad (15\%)$$

$$u(0, t) = u(\pi, t) = 0, (t \geq 0)$$