國立中央大學八十四學年度碩士班研究生入學試題卷

所別：物理研究所　　組　　科目：近代物理學　　共 一 頁 第 一 頁

PLEASE READ THIS SHORT MESSAGE FIRST:
Please work out the following problems in detail, otherwise put down how you may proceed. Attempt as many problems as you can but spend your time *wisely*. Please pay attention to the score of each problem. Good luck!

(1) (25 points) Write down the time dependent Schrödinger equation with Hamiltonian $\hat{H}$. What is the definition of the expectation value of an observable $\hat{O}$? Derive an equation governing the time development of the expectation value of $\hat{O}$. Find the conditions under which the expectation value of $\hat{O}$ does not change with time.
A particle moving non-relativistically in a one dimension symmetric potential $V(x) = V(-x)$. Does the expectation value of the parity operator change with time? Give reasons to support your answer.
Write down the set of normalised eigenstates of a particle in a box with walls at $x = -L$ and $x = L$. Now suppose $L = 1/2$ and the particle is initially ($t = 0$) at the state

$$\psi(0, x) = \frac{\sqrt{2}}{5}\left[4\sin(4\pi x) - 3\cos(3\pi x)\right]$$

What is its state at time $t > 0$? Find the expectation value of the parity at $t = 0$ and $t > 0$.

(2) (25 points) From the time dependent Schrödinger equation, derive the continuity equation for the probability density $\psi^*\psi$ and identify the probability current density (or flux).
A particle of energy E is moving in a step potential $V(x)$, where $V(x) = 0$ if $x < 0$ and $V(x) = V_0$ if $x \geq 0$. If $0 < E < V_0$, compute the wavefunction of the particle. State clearly the boundary conditions used. If this system is interpreted as a scattering, what is the reflection coefficient? Sketch the wavefunction as a function of x. Is it possible to observe the particle in the region $x > 0$? If yes, how? If no, why?

(3) (25 points) The fine structure energy of a hydrogen atom derived from the first order time independent perturbation theory of quantum mechanics is (assume that the nucleus is infinitely heavy)

$$E = -\frac{m_e c^2 \alpha^2}{2n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{2n}{2j+1} - \frac{3}{4}\right)\right]$$

where $\alpha = e^2/(4\pi\epsilon_0 \hbar c)$. What is the small parameter used in the perturbation? What are the quantum numbers n and j? What values can j takes? Write down the ground state energy.
Now the relativistic ground state energy of the electron in the hydrogen atom can be estimated as follows. First, write down the total energy of the electron in relativistic form. Then assume that the radius r and momentum p are related by $r\,p = \hbar$. Give a reason to support this assumption. Now compute the minimum energy (i.e., the ground state energy) of the electron. Does this result agree with the result from the quantum mechanical calculation?

(4) Answer the following questions briefly.
  (a) (5 points) What are the basic postulates of Einstein's theory of special relativity? Describe a phenomenon about it.
  (b) (5 points) Describe the phenomenon of quantum tunneling. Give an example of this process. Is there an analogy of quantum tunneling in classical physics? If yes, give an example.
  (c) (5points) What is the meaning of identical particles in quantum mechanics? What is the different between Fermion and Boson? Write down the singlet and triplet spin eigenstates of a two electron system.
  (d) (5 points) Interacting with radiation, an atom may jump from one energy state to another. What are the different ways of transition? Describe the principle of laser in a few words.
  (e) (5 points) Up to now, how many fundatmental forces have been found? Describe their characteristics in a few words.