所別：物理學系碩士班 不分組 科目：應用數學

You **must** show the steps clearly in order to earn credits.

1. (10%) A particle moves in the x-y plane so that its position (x,y) as a function of time t, when is expressed using a complex form, is giving by
$$z \equiv x+iy = \frac{i+2t}{t-i} \quad \text{where} \quad i \equiv \sqrt{-1}.$$
Find the magnitudes of its velocity and its acceleration as functions of t.

2. (10%) Evaluate the surface integration $\oint \vec{V} \cdot \hat{n} dS$ of a vector function, representing in rectangular cartesian coordinates, $\vec{V} = (x, 2y, 3z)$ over the surfaces of a cylinder of height h and radius a.

3. (10%) Solve the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$.

4. (15%) Find the eigenvalues and their corresponding normalized eigenvectors of $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

5. (15%) Evaluate the integral $\int_0^{\pi} \frac{d\theta}{1+\sin^2\theta}$ by means of setting $Z = e^{i\theta}$.

6. (20%) The vertical displacement y of an evenly loaded, two ends fixed (兩端固定) beam of length L may be described by using the differential equation $\frac{d^4y}{dx^4} = \alpha_0 = const.$ (a) Expanding α_0 in Fourier series; (b) then find $y(x)$.

7. (20%) Consider the differential equation $\frac{\partial^2 g(x,t)}{\partial x^2} = \delta(x-t)$, where $\delta(x-t)$ is the Dirac delta function, is subjected to boundary conditions of $g(0,t) = 0$ and $g(1,t) = 0$. (a) Obtaining the general solutions for $g(x,t)$ in the $x < t$ and in the $x > t$ regimes; and (b) combining them to obtain $g(x,t)$ for all x.