Fig. 1. Let A, B, C be three measurable events in a probability space. Suppose that A and B are independent and P(A)=0.6, P(B)=0.7, P(C B)=0.2 and P(C AB)=0.1. Let A <sup>2</sup> be the complement of A, Find (1) P(A B) (5%) (2) P(BC A) (5%) (3) P(C A C) (5%) (4) P(A^C B C) (5%) (5%) (1) Show that (Cov(X,Y)) <sup>2</sup> ≤ Var(X) Var(Y) (5%) (2) Show that for arbitrary measurable events A <sub>1</sub> , A <sub>2</sub> ,, A <sub>n</sub> in a probability space, the following inequality holds:  P(A <sub>1</sub> A <sub>2</sub> A <sub>n</sub> ) ≥ 1 - \frac{1}{2} P(A_1^C). (5%) (2) Use (1) to prove the Chebyshev inequality. (5%) (3) Suppose that \(\frac{1}{2}\) for (and as a binomial distribution b(n,p) for n=1, 2 Show that S <sub>d</sub> /n converges to p in probability as n→∞ by using the Chebyshev inequality. (5%)  IV. If f and F be the pdf and cdf of an absolutely continuous distribution. For n≥2, define \( f_n(y_1, y_2,, y_n) = f(y_n) \) \frac{1}{11} \frac{1}{1} f(y_i)/(1-F(y_i)),  y_1 \leq y_2 \leq \leq y_n \) \( = 0, \text{ otherwise.} \)  (1) Suppose  (1) Suppose  (1) Suppose  (1) Suppose  (2) Determine the distribution of Y <sub>n</sub> using the information in (1). (5%) (3) Show that Y <sub>n</sub> is a symptotic normal and determine the norming constants. (5%)  V. Let (X,Y) have the following joint pdf \( f(x,y) = a \) for OSySex1 or OSx≤1/2, x+1/2Sy≤1 \( = 0 \) otherwise.  Find (1) a (5%) (2) Marginal pdf of Y (5%) (3) E[XY Y=1] (5%) (3) E[XY Y=1] (5%) (4) P(2X+Y \leq 3/2) (5%) (5%) (5%) (5%)	國立中央大學八十五學年度碩士班研究	完生入學試題卷
<ol> <li>Let A, B, C be three measurable events in a probability space. Suppose that A and B are independent and P(A)=0.6, P(B)=0.7, P(CIB)=0.2 and P(CIAB)=0.1. Let A<sup>C</sup> be the complement of A. Find (1) P(A(B) (5%) (2) P(BCIA) (5%) (3) P(CIA<sup>C</sup>B) (5%) (4) P(A<sup>C</sup>BC) (5%) (3) P(CIA<sup>C</sup>B) (5%) (4) P(A<sup>C</sup>BC) (5%) (4) P(A<sup>C</sup>BC) (5%) (5%) (2) Show that [Cov(X,Y)]<sup>2</sup> ≤ Var(X) Var(Y) (5%) (2) Show that for arbitrary measurable events A<sub>1</sub>, A<sub>2</sub>,, A<sub>n</sub> in a probability space, the following inequality holds:</li></ol>	所別:統計研究所 甲組 科目:機率論	共 <u>/</u> 頁 第 <u>/</u> 頁
<ul> <li>(2) P(BCA) (5%)</li> <li>(3) P(CAPB) (5%)</li> <li>(4) P(APBC) (5%)</li> <li>(4) P(APBC) (5%)</li> <li>(5%)</li> <li>(4) P(APBC) (5%)</li> <li>(5%)</li> <li>(2) Show that [Cov(X,Y)]<sup>2</sup> ≤ Var(X) Var(Y) (5%)</li> <li>(2) Show that for arbitrary measurable events A<sub>1</sub>, A<sub>2</sub>,, A<sub>n</sub> in a probability space, the following inequality holds:     P(A<sub>1</sub>A<sub>2</sub>A<sub>n</sub>) ≥ 1- ∑ P(A<sub>1</sub><sup>2</sup>). (5%)</li> <li>(5%)</li> <li>(111. (1) Let g(x) &gt; 0 for x&gt;0 be a monotonically increasing function. Suppose F[g(XI)] = M exists. Show that P(XI≥x) ≤ M/g(x) for all x&gt;0. (5%)</li> <li>(2) Use (1) to prove the Chebyshev inequality. (5%)</li> <li>(3) Suppose that S<sub>n</sub> follows a binomial distribution b(n,p) for n=1, 2,</li> <li>Show that S<sub>n</sub>/n converges to p in probability as n→∞ by using the Chebyshev inequality. (5%)</li> <li>(3%)</li> <li>(4) If f and F be the pdf and cdf of an absolutely continuous distribution. For n≥2, define f<sub>n</sub>(y<sub>1</sub>, y<sub>2</sub>,, y<sub>n</sub>) = f(y<sub>n</sub>) ∏ f<sub>1</sub>(y<sub>1</sub>)/11-F(y<sub>1</sub>), y<sub>1</sub>≤ y<sub>2</sub>≤≤ y<sub>n</sub> = 0, otherwise.</li> <li>(1) Suppose</li> <li>(1) Suppose</li> <li>(1) Suppose</li> <li>(2) Otherwise</li> <li>(3) On therwise</li> <li>(4) and y<sub>1</sub> y<sub>2</sub> y<sub>2</sub> y<sub>3</sub> are the n random variables with f<sub>n</sub> as their joint pdf. Define W<sub>1</sub> y<sub>1</sub> y<sub>1</sub> y<sub>1</sub> y<sub>1</sub> y<sub>2</sub> y<sub>2</sub> y<sub>3</sub> y<sub>n</sub> are independant.</li> <li>(2) Determine the distribution of Y<sub>n</sub> using the information in (1). (3%)</li> <li>(3) Show that Y<sub>n</sub> is asymptote normal and determine the norming constants. (3%)</li> <li>V. Let (X,Y) have the following joint pdf f(x,y) = a for 05y≤x≤1 or 05x≤1/2, x+1/2≤y≤1 = 0 otherwise.</li> <li>Find</li> <li>(1) a (5%)</li> <li>(2) Marginal pdf of Y (5%)</li> <li>(3) E[XY-y] (5%)</li> <li>(4) P(2X+Y≤3/2) (5%)</li> <li>(5%)</li> </ul>	<ol> <li>Let A, B, C be three measurable events in a probability space. Suppose that are independent and P(A)=0.6, P(B)=0.7, P(C B)=0.2 and P(C AB)=0.1.</li> </ol>	A and B
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<ul> <li>(3) Suppose that S<sub>n</sub> follows a binomial distribution b(n,p) for n=1, 2, Show that S<sub>n</sub>/n converges to p in probability as n→∞ by using the Chebyshev inequality. (5%)</li> <li>IV. If f and F be the pdf and cdf of an absolutely continuous distribution. For n≥2, define f<sub>n</sub>(y<sub>1</sub>, y<sub>2</sub>,, y<sub>n</sub>) = f(y<sub>n</sub>) ∏ f(y<sub>i</sub>)/[1-F(y<sub>i</sub>)], y<sub>1</sub>≤ y<sub>2</sub>≤≤ y<sub>n</sub> = 0, otherwise.</li> <li>(1) Suppose f(y) = e<sup>-y</sup>, y≥0 = 0, otherwise and Y<sub>1</sub>, Y<sub>2</sub>,, Y<sub>n</sub> are the n random variables with f<sub>n</sub> as their joint pdf. Define W<sub>i</sub> = Y<sub>i</sub> - Y<sub>i-1</sub>, i=1, 2,, n, with Y<sub>0</sub> = 0. Determine whether W<sub>1</sub>, W<sub>2</sub>,, W<sub>n</sub> are independant. (10%)</li> <li>(2) Determine the distribution of Y<sub>n</sub> using the information in (1). (5%)</li> <li>(3) Show that Y<sub>n</sub> is asymptotic normal and determine the norming constants. (5%)</li> <li>V. Let (X,Y) have the following joint pdf f(x,y) = a for 0≤y≤x≤1 or 0≤x≤1/2, x+1/2≤y≤1 = 0 otherwise.</li> <li>Find (1) a (5%)</li> <li>(3) E[X1Y=y] (5%)</li> <li>(4) P(2X+Y≤3/2) (5%)</li> <li>VI. Let X have a pdf f(x) which is a mixture of two pdfs f<sub>1</sub> and f<sub>2</sub> as follows: f(x) = (1-e) f<sub>1</sub>(x) + ε f<sub>2</sub>(x), 0&lt;ε&lt;1.</li> <li>Suppose now f<sub>1</sub>(x) is the pdf of a normal distribution with mean μ<sub>1</sub> and variance σ<sup>2</sup><sub>1</sub>, i=1, 2. Find</li> <li>(1) E[X] (5%)</li> <li>(2) Var(X)</li> </ul>	-	
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