

科目：數值分析(1003)

校系所組：中大數學系乙組

參考用

Instructions: Do all 5 problems. Show your work.

1. (Nonlinear equation)

- (a) Suppose we approximate  $\sqrt{5}$  by using the secant method to solve  $x^2 - 5 = 0$ . Give a formula for  $x_{n+1}$  in terms of  $x_n$  and  $x_{n-1}$ . (Simplify the formula as much as possible) (10pts)
- (b) Start  $x_0 = 2$  and  $x_1 = 3$  find  $x_2$ . (5pts)

2. (Numerical linear algebra)

- (a) Give a formal definition of a diagonally dominant matrix  $A$ . (10 pts)
- (b) Determine whether the following matrix is diagonally dominant or not. Give your reason.

$$A = \begin{pmatrix} 3+i & 2 & 1 \\ 1 & 2 & 0 \\ 2 & 1 & 1+2i \end{pmatrix}$$

(10 pts)

3. (Numerical Differential Equations)

- (a) Assume that  $u \in C^4[x_0 - h, x_0 + h]$ . Use Taylor's theorem to derive the second-order central difference approximation for the second derivative of  $u(x)$  at  $x = x_0$ , including the error term, for  $h > 0$ :

$$u''(x_0) = \frac{u(x_0 + h) - 2u(x_0) + u(x_0 - h)}{h^2} - \frac{h^2}{12} u^{(4)}(\xi), \quad (1)$$

where  $\xi \in (x_0 - h, x_0 + h)$ . (10 pts)

- (b) Solve the following two-point boundary value problem:

$$\begin{cases} u'' + 4x = 0 \\ u(0) = 1 \quad u(1) = 0 \end{cases}$$

for  $u(1/2)$  approximately using Formula (1) with  $h = 1/2$ . (10 pts)

注意：背面有試題

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4. (Numerical Integration)

- (a) Construct a rule of the form on  $[-1, 1]$

$$\int_{-1}^1 f(x) \approx Af(-1/2) + Bf(0) + Cf(1/2)$$

that is exact for all polynomials of degree  $\leq 2$ ; that is, determine values for  $A, B$ , and  $C$ . (10 pts)

- (b) Write down the corresponding rule for an arbitrary interval  $[a, b]$ . (10 pts)

5. (Interpolation)

Consider the table

$x$	1	3	2	4
$y$	-1	-3	-2	1

- (a) Find the Lagrange form of the interpolation polynomial of degree 3 passing the points given in the above table. (10 pts)
- (b) Redo part (a) by using the Newton form of the interpolation polynomial. (10 pts)
- (c) Approximate  $f''(x)$  at  $x = 5$ . (5pts)