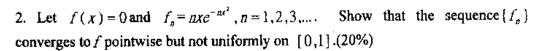
國立中央大學八十六學年度碩士班研究生入學試題卷

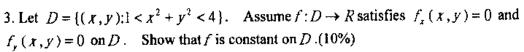
所別: 數學研究所 不分組 科目:

高等微積分

共 1 頁 第 1 頁

1. Let D be a bounded subset of the set of real numbers R. Show that $f:D\to R$ is bounded on D if and only if for each $x\in R$, there is a neighborhood N_x of x such that f is bounded on $D\cap N_x$. (20%)







4. Let $D = \{(x, y): x > 0, y > 0, 1 \le xy \le 3, 1 \le x^2 - y^2 \le 7\}$. Evaluate $\iint_{\mathcal{S}} x^2 + y^2 dx dy . (10\%)$

5. Show that
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$$
 for all x in $(-1,1].(20\%)$

6. Suppose $f: \mathbb{R}^2 \to \mathbb{R}$ and $g: \mathbb{R}^2 \to \mathbb{R}$ are both continuous on \mathbb{R}^2 , differentiable at (0,0), and $f_x(0,0) = g_y(0,0)$. Show that for each $\varepsilon > 0$, there is a $\delta > 0$ such that $\left| \oint_{\mathbb{R}^2 + y^2 = r^2} f(x,y) dy + g(x,y) dx \right| < \varepsilon r^2$ whenever $0 < r < \delta$. (20%)