1. Let $A=\begin{bmatrix}1&2&3\\3&1&2\\2&-2&1\end{bmatrix}$, $B=\begin{bmatrix}0&1&1\\1&0&1\\-1&2&2\end{bmatrix}$, $C=\begin{bmatrix}1&1&1\\1&-1&0\\1&-1&-1\end{bmatrix}$. Compute the following values. (25%)

(i) A^{-1}, (ii) det(AB), (iii) det(BC), (iv) trace(A+B+C), (v) eigenvalues of A.

2. Let P be the plane in R^3 defined by $P=\{(x,y,z): x+y+z=0\}$. Let T be the reflection with respect to P; i.e. for each v in R^3, $T(v)$ is the mirror image of v on the other side of P. Find eigenvalues and eigenvectors of T. Represent T as a matrix in two ways; one with respect to eigenvectors as a basis and the other with respect to the standard basis. (15%)

3. Give a condition on b_1, b_2, b_3, b_4 so that the following system of linear equations is solvable. (15%)

$$\begin{cases} x + 2y = b_1 \\ 2x + 4y = b_2 \\ 2x + 5y = b_3 \\ 3x + 9y = b_4 \end{cases}$$

4. Let A be an $n \times m$ matrix. Show that A has linearly independent columns if and only if $A^T A$ is invertible, where A^T is transpose of A. (15%)

5. Let $A=\begin{bmatrix}1&-1&2&-2&3\\2&-2&4&-4&6\\3&-3&6&-6&9\\0&4&5&2&1\end{bmatrix}$. Find a basis for each of the following spaces; (i) row space of A, (ii) column space A, (iii) nullspace of A, (iv) null space of A^T which is transpose of A. (15%)

6. Let $A=\begin{bmatrix}3&0&1\\0&4&0\\1&0&3\end{bmatrix}$. Find a diagonal matrix Λ and an orthogonal matrix Q such that $\Lambda = Q^{-1}AQ$. (15%)