

- 一. Show that every group G with identity e and such that $x*x = e$ for all $x \in G$ is abelian. (10%)
- 二. If A is a set, then a subgroup H of the permutation group (symmetric group) S_A is transitive on A if for each $a, b \in A$ there exists $\sigma \in H$ such that $\sigma(a) = b$. Show that if A is a nonempty finite set, then there exists a finite cyclic subgroup H of S_A with $|H| = |A|$ that is transitive on A . (10%)
- 三. Let \mathbb{Z}_m be the group of residue classes modulo m . Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to \mathbb{Z}_{mn} if and only if m and n are relatively prime. (10%)
- 四. Show that every group of order 33 is cyclic. (10%)
- 五. Prove that $25x^5 - 9x^4 + 3x^2 - 12$ is irreducible over \mathbb{Q} = {rational numbers}. (10%)
- 六. Show that a commutative ring with unity is a field if and only if it has no proper nontrivial ideals. (10%)
- 七. Prove that $\mathbb{Z}[\sqrt{-5}] = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}, \text{ integers}\}$ is an integral domain but not a unique factorization domain. (10%)
- 八. Show that if F is any finite field, then for every positive integer n , there is an irreducible polynomial in $F[x]$ of degree n . (10%)
- 九. Prove that if E is a finite separable extension of a field F , then there exists $\alpha \in E$ such that $\bar{E} = F(\alpha)$. (10%)
- 十. Find the Galois group of $2x^5 - 10x + 5$ over \mathbb{Q} . (10%)