國立中央大學九十二學年度碩士班考試入學招生試題卷

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系所別： 數學系　　科目： 機率與統計

參考用

(36%) 1. Let X, Y be independent random variables taking values in N with $P(X=i) = P(Y=i) = \frac{1}{2^i}$, $i = 1, 2, 3, \cdots$

Find (a) $P(\min(X,Y) \le i)$

(b) $P(X = Y)$

(c) $P(X > Y)$

(d) $P(X \text{ divides } Y)$

(e) $P(X \ge kY)$, $k \in N$

(f) $P(X+Y > 10)$

(10%) 2. Let X, Y be two independent discrete random variables. Suppose $P(X+Y=\alpha) = 1$, where α is a constant. Show that both X and Y are constant variables.

(18%) 3. Let U be a random variable distributed uniformly on $(0,1)$. Let $Y = \min\{U, 1-U\}$

(a) Find the p.d.f. of Y

(b) Find EY and $\mathrm{Var}\, Y$.

(18%) 4. Suppose that $U_1, U_2, \cdots$ are independent $U(0,1)$ random variables. Let N be the first $n \ge 2$, such that $U_n \ge U_{n-1}$.

(a) Show that $P(U_1 > U_2 > U_3) = \frac{1}{6}$

(b) $P(U_1 \le u, N = n) = \frac{u^{n-1}}{(n-1)!} - \frac{u^n}{n!}$

(c) Find EN.

(9%) 5. Find the Neyman-Pearson size α test of $H_0 : \beta = 1$ against $H_1 : \beta = \beta_1 (>1)$, based on a sample of size 1 from

$$f(x;\beta) = \begin{cases} \beta x^{\beta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(9%) 6. Let X be a Bernoulli trial with parameter p. $p \in [\frac{1}{4}, \frac{3}{4}]$. Show that the MLE of p is $\frac{2X+1}{4}$.