# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 統計 共 2 頁 第 1 頁

- SHOW YOUR WORK & GOOD LUCK !
- Use Table 1 & Table 2 if necessary

1. (20%) Given a coin. Let $p$ be the probability that the head appears. We are interested in estimating the unknown parameter $p$.
   (a). (10%) Construct a statistical model and then give 2 estimators for $p$.
   (b). (10%) Compare the 2 estimators in (a). Which one is better ? Why ?

2. (10%) Let $Y_1, Y_2, \ldots, Y_6$ be a random sample from a normal population with a mean 0 and a variance of 1. Let $\bar{Y} = \frac{1}{5}\sum_{i=1}^{5} Y_i$ and let $U = \sum_{i=1}^{5}(Y_i - \bar{Y})^2$. What is the distribution of $\frac{2(5\bar{Y}^2+Y_6^2)}{U}$ ? Why ?

3. (15%) If $Y_1, Y_2, \ldots, Y_n$ denotes a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$.
   (a). (10%) If $\mu$ is unknown and $\sigma^2$ is known, show that the sample mean $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is sufficient for $\mu$. ( State the definition or theorem that you need. )
   (b). (5%) If $\mu$ is known and $\sigma^2$ is unknown, find the method of moment estimator for $\sigma^2$.

4. (15%) Suppose $Y_1, Y_2, \ldots, Y_n$ denotes a random sample from a uniform distribution with a probability density function, given by

$$f(y|\theta) = \begin{cases} \frac{1}{2\theta+1}, & 0 \le y \le 2\theta+1 \\ 0, & \text{otherwise} \end{cases}$$

   Find the maximum-likelihood estimator of $\theta$. Justify your answer.

5. (20%) Shear strength measurements derived from unconfined compression test for two types of soils gave the results shown in the following table ( measurements in ton per square foot )

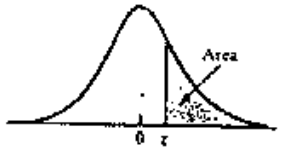
| Soil Type I | Soil Type II |
|---|---|
| $n_1 = 30$ | $n_2 = 35$ |
| $\bar{y}_1 = 1.65$ | $\bar{y}_2 = 1.43$ |
| $s_1 = 0.26$ | $s_2 = 0.22$ |

   ( Note that if $Y_1, \ldots, Y_n$ is a random sample then sample mean $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$. )
   (a). (10%) Do the data present sufficient evidence to suggest a difference between the true mean shear strength for the above two types of soils ? Use type I error $\alpha = .01$.
   (b). (10%) State the assumption(s) and theorem(s) you need to solve (a).

6. (20%) The Environmental Protection Agency has collected data on LC50 measurements ( concentrations that kill 50 % of test animals ) for certain chemicals likely to be found in freshwater rivers and lakes. For certain species of fish, the LC50 measurements for DDT in six experiments are as follows :
   12, 8, 5, 2, 9, 6
   (a). (10%) Estimate the mean LC50 for DDT, with confidence coefficient .90.
   (b). (10%) State the assumption(s) and theorem(s) you need to solve (a).

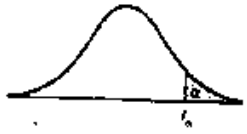
Tables

**Table 1**
Normal curve areas
Standard normal probability in right-hand tail (for negative values of z areas are found by symmetry)




| | *Second decimal place of z* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *z* | *.00* | *.01* | *.02* | *.03* | *.04* | *.05* | *.06* | *.07* | *.08* | *.09* |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| 1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| 2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| 2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| 2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| 2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| 2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| 2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| 3.0 | .00135 | | | | | | | | | |
| 3.5 | .000 233 | | | | | | | | | |
| 4.0 | .000 031 7 | | | | | | | | | |
| 4.5 | .000 003 40 | | | | | | | | | |
| 5.0 | .000 000 287 | | | | | | | | | |

**Table 2**
Percentage points of the t distribution




| $t_{.100}$ | $t_{.050}$ | $t_{.025}$ | $t_{.010}$ | $t_{.005}$ | d.f. |
|---|---|---|---|---|---|
| 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 |
| 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | inf. |