

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 複變函數論 共 1 頁 第 1 頁

All answers must be justified and work must be shown.

- 20% 1. Let $f: \mathbb{C} - \{0, 1\} \rightarrow \mathbb{C}$ be an analytic function on the complex plane except the two points 0 and 1 such that $f(z) = \sum_{n=-\infty}^{\infty} a_n z^n$ for $|z| > 1$, where $a_n = 1$ for $n < 0$ and $a_n = 1/n!$ for $n \geq 0$. Determine what type of singularity f has at 0, 1 and ∞ .

- 20% 2. Find the smallest integer n such that there is no $z \in \mathbb{C}$ with

$$z^{11} + z^5 + 40z + 1999 = 0$$

and $|z| \geq n$. Explain. ($3^{11} = 177147$, $3^5 = 243$, $2^{11} = 2048$, $2^5 = 32$.)

- 20% 3. Evaluate the integral

$$\int_0^{\infty} \frac{\sqrt{x}}{x^2 + 1} dx.$$

- 20% 4. Does there exist an analytic function f in the unit disc $\{z: |z| < 1\}$, with the following property

$$f(1/n) = \frac{n}{2+n} \text{ for } n = 1, 2, 3, \dots?$$

Give an example or prove that it does not exist.

- 20% 5. Find a conformal mapping of the region $\{z: |z| < 1, \operatorname{Im} z > 0\}$ onto the disc $\{z: |z| < 1\}$. If you use a composition of maps, you need only indicate the individual components.