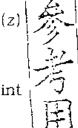
(20%) 1. If u(z) is nonconstant and harmonic in the plane, show that u(z) comes arbitrarily close to every real value.



- (20%) 2. If $\alpha > 1$, prove that $f(z) = z + e^{-z}$ takes the value α at exactly one point in the right half-plane.
- (20%) 3. Does there exist a function f(z) analytic in |z| < 1 and satisfying

$$f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{2n}$$
 $\{n = 1, 2, \ldots\}$?

- (20%) 4. If f(z) is an entire function and $|f(z)| \le Mr^{\lambda}$ ($|z| = r \ge r_0$) for some nonnegative real number λ . Prove that f(z) is a polynomial of degree at most λ .
- (10%) 5. Compute the integral

$$\int_{|z|=\rho} \frac{|dz|}{|z-\alpha|^2}$$

under the condition $|a| \neq \rho$.

(10%) 6. Determine the LFT S(z) with $S(i) = \infty$, S(0) = 0 and S maps $\text{Im } z \ge 0$ onto $\{|w-1| \ge 1\}$.