

國立中央大學98學年度碩士班考試入學試題卷

所別：生物醫學工程研究所碩士班 科目：工程數學 共 2 頁 第 1 頁

所別：機械工程學系碩士班 甲組(固力與設計)

*請在試卷答案卷(卡)內作答

所別：機械工程學系碩士班 乙組(製造與材料)

所別：機械工程學系碩士班 丙組(熱流)

所別：光機電工程研究所碩士班 乙組(光機組)

所別：能源工程研究所碩士班

Ordinary Differential Equation (25 %)

1. A $147N$ weight is attached to the lower end of a spring suspended from the ceiling. The weight comes to rest in its equilibrium position thereby stretching the spring $0.6m$. The weight is then pulled down $0.2m$ below its equilibrium position and released at $t=0$. No external forces are present but the medium offers a resistance numerically equal to $4dx/dt$, where dx/dt is the instantaneous velocity in m/sec . Show the differential equation which governs the resulting motion of the weight on the spring. (5%)

2. Consider the equation $2t^3y'' + t^2y' + y = 0$ for large values of t .

(a) Give the values of m if the solution is assumed to be

$$y = \left(\frac{1}{t}\right)^m \sum_{n=0}^{\infty} a_n \left(\frac{1}{t}\right)^n \quad (10\%)$$

(b) Determine the recursion formula and compute a_1 and a_2 in terms of a_0 . (5%)

3. Solve the differential equation $y'' + (y')^2 + 1 = 0$. (5%)

Complex Analysis (25 %)

4. Given a complex function $f(z) = \cosh(z)$

(a) Determine the value of $f(z)$ at $z = -2i$ where i is defined as $\sqrt{-1}$. (4%)

(b) Find the value of $\left. \frac{df(z)}{dz} \right|_{z=-2i}$. (4%)

(c) Find all the singularities of the two complex functions $g_1(z) = \frac{3z^2}{(z+2i)^2}$ and

$g_2(z) = \frac{\cosh(z)}{z^4}$, respectively. Identify the order of each singularity/pole or indicate that the singularities are essential singular points. (7%)

(d) Evaluate the integral $\int_{\Gamma} \frac{3z^2 \cosh(z)}{(z+2i)^2} dz$, where Γ is the circle of radius 8 about 1. (10%)

參考用

注意：背面有試題

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Vector Calculus (25 %)

5. Find the surface integral of the vector function $\mathbf{F} = [y^3, x^3, z^3]$ over the portion of the surface defined as $S: x^2 + 4y^2 = 4, x \geq 0, y \geq 0, 0 \leq z \leq h$. (25%)

Partial Differential Equation (25 %)

6. Solve the partial differential equation (7%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0$$

$$u(x, 0) = \exp(-|x|), \quad \frac{\partial u(x, 0)}{\partial t} = 0, \quad -\infty < x < \infty.$$

7. Solve the partial differential equation (18%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

$$u(x, 0) = x, \quad 0 < x < 1$$

$$u(0, t) - 3 \frac{\partial u(0, t)}{\partial x} = 1, \quad u(1, t) = 5, \quad t > 0.$$

參考用

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