國立中央大學八十八學年度碩士班研究生入學試題卷

所別：機械工程研究所甲乙丙組　科目：甲.乙.丙工程數學　共 2 頁 第 1 頁

1. Solve the following problems

(1) Let $A = (a_{ij})$ be an $n \times n$ matrix such that for each $i = 1,\dots n$

we have

$$\sum_{j=1}^{n} a_{ij} = 0$$

Show that 0 is an eigenvalue of A. (8%)

(2) A recursive formula is given as

$$r_{n+1} = 4r_n - t_n, \quad t_{n+1} = 2r_n + t_n$$

with the initial values, $r_0 = 100$ and $t_0 = 10$. Determine

$\lim_{n\to\infty} \frac{r_n}{t_n} = ?$ (8%)

(3) A nonhomogeneous system of equations is given as

$$x - 2y + 3z = 1$$
$$2x + ky + 6z = 6$$
$$-x + 3y + (k-3) = 0$$

Determine the value of k for which (a) the system has a unique solution (3%), (b) the system has no solution (3%), and (c) the system has general solution (3%).

參考用

2. Solve the following partial differential equation:

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial x^2} + 1$$

$$\theta(x,0) = 0$$

$$\frac{\partial\theta(0,t)}{\partial x} = 0$$

$$\theta(L,t) = 0$$

Please solve the problem according to the following the steps:

(1) Separation of variables: $\theta(x,t) = \psi(x,t) + \varphi(x)$. (5%)

(2) Solve $\varphi(x)$. (5%)

(3) The variable $\psi(x,t)$ needs a further separation of variable. (5%)

(4) Solve $\psi(x,t)$ and get the full answer of $\theta(x,t)$. (10%)

3. (a) (10%) Find the eigenvalues and eigenvectors of A, where A is defined as follows.

$$A = \begin{pmatrix} -6 & -4 & -2 \\ -4 & -6 & -2 \\ -2 & -2 & -17 \end{pmatrix}$$

(b) (5%) Find a diagonal matrix D and an orthogonal matrix P, such that

$$D = P^T A P$$

(c) (10%) Find the solution $x(t)$ for a linear system,

$$\frac{dx(t)}{dt} = Ax(t)$$

with its initial condition $x(0) = (2, 4, 3)^T$.

注

4. The governing equation for a uniform beam on an elastic foundation

can be written by: $\frac{d^4 y}{dx^4} + \beta^4 = 0$, *where* : $\beta^4 = \frac{k}{4EI}$,

y is the deflection of the beam. EI, k are constants.

(a) Find the general solution of this equation. (10%)

(b) A semi-infinite beam with concentrated loads, P and M, act at the end x = 0. The deflection y must vanish at x = ∞, and the conditions at the origin are:

$$EI\left(\frac{d^2 y}{dx^2}\right)_{x=0} = -M, \qquad EI\left(\frac{d^3 y}{dx^3}\right)_{x=0} = P,$$

Find the solution of deflection curve y. (15%)

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