

國立中央大學八十四學年度碩士班研究生入學試題卷

所別：機械工程研究所

組

科目：工程數學

共 1 頁 第 1 頁

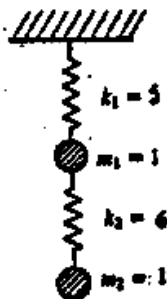
參考用

1. Consider the following Sturm-Liouville problem:
 $(xy')' + (\lambda/x)y = 0, y'(1) = 0, y'(2) = 0.$
 Find the eigenvalues and eigenfunctions. (15%)

2. Consider the following equation
 $x^2 y'' + xy' + (\lambda^2 x^2 - \nu^2)y = 0.$
 Find a general solution of the given equation in terms of Bessel functions. (10%)

3. Find the standard form of the quadratic polynomial
 $3x_1^2 + 3x_2^2 - 4x_1x_2 - 3x_1x_3 + 2x_2x_3 + x_3^2$ (10%)

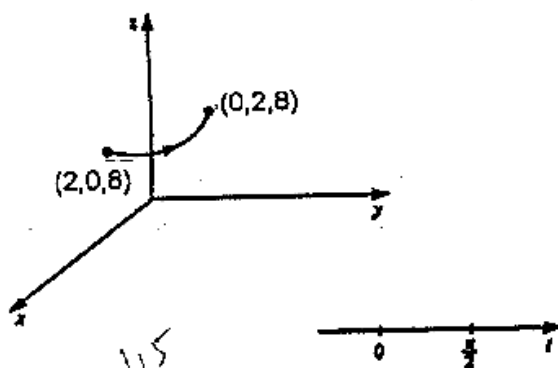
4. (1) Set up the equations of motion of the spring system shown below.
 (2) Find out eigenvalues and eigenvectors of the corresponding system.
 (3) Write down the general solution.
 (4) Describe the physical meaning of the eigenvectors for the given system. (15%)



5. (1) 試推導弦之振動方程式，並說明相關假設。(10%)
 (2) 考慮空氣阻尼時(或其它介質阻尼時)方程式有何需要修正。(8%)
 (3) 說明相關 eigenvalue 及 eigenmode 的物理涵義。(7%)

6. Find the work done by $F = i - yj + xyzk$ as it moves an object along the curve $R(t) = ti - t^2j + tk$ from $(0,0,0)$ to $(2,-4,2)$. (10%)

7. A wire is bent into the shape of a quarter-circle whose parametric equations are $x = 2 \cos(t), y = 2 \sin(t), z = 8$ for $0 \leq t \leq \pi/2$. The wire is shown below. The density function is $\delta(x,y,z) = xy$ grams/centimeter. Find the mass and center of mass. (15%)



115