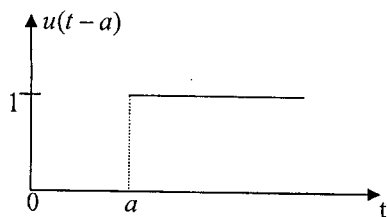所別：土木工程學系碩士班 結構組　科目：工程數學　共＿1＿頁 第＿1＿頁

1.

(a)　請問以下之函數集合 $\{ x^2, \ x|x|, \ x\}$ 在區間 $x > 0$ 是綫性獨立或是綫性相依？

(5%)

(b)　請求以下函數 $u(t-a)$ 之微分，$\dfrac{d}{dt}[u(t-a)] = ?$

(10 %)

2.　請求解 $y' + p(x)y = r(x)$

(25 %)

3)　Consider a periodic function $f(t)$.　In the interval $-1 \le t \le 1$ the function is described by $f(t) = t^2$. If the Fourier series of this periodic function is written as $f(t) = \dfrac{1}{3} + \sum\limits_{n=1}^{\infty} a_n \cos(n * \pi * t)$，please find out the coefficient a_n.

(25%)

4)　Let $A = (A_{ij}) = \begin{pmatrix} 4 & -5 \\ 1 & -2 \end{pmatrix}$ be a 2×2 matrix and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. If A satisfies the identity $A^3 = aA + bI$ please find out the real numbers a and b.

(25%)

5)　A curve is represented by the following position vector

$$\vec{r} = [\cos(t) + t * \sin(t)]\,\vec{i} + [\sin(t) - t * \cos(t)]\,\vec{j} + [t^2]\,\vec{k},$$

where t is a parameter. At a point P on the curve the position vector is

$\vec{r} = \dfrac{\pi}{2}\vec{i} + \vec{j} + \dfrac{\pi^2}{4}\vec{k}$. Please compute the curvature of this curve at the point P.

(10%)