

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 土木工程學系甲丙戊組 科目: 工程數學 共 1 頁 第 1 頁

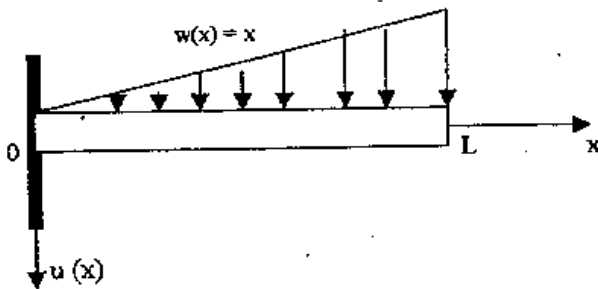
1)

如下圖所示之彎曲剛度為 EI 之懸臂梁，在 $0 < x < L$ ，承受一荷載 $w(x) = x$ ，當 $x > L$ 時 $w(x) = 0$ 。此一梁撓曲變形 $u(x)$ 之控制方程為

$$EI \frac{d^4 u}{dx^4} = w(x)$$

$$\text{邊界條件為 } u(0) = \frac{du}{dx}(0) = \frac{d^2 u}{dx^2}(L) = \frac{d^3 u}{dx^3}(L) = 0$$

請用 Laplace 轉換求解此一梁之撓曲變形 $u(x) = ?$ (25%)



2) 已知一個矩陣 $[A]$ 的三個特徵值分別為 $1, 2, 3$ ，請求出下式中的 α, β, γ 之值。(25%)

$$[A]^4 = \alpha[A]^2 + \beta[A] + \gamma[I]$$

3) Consider the following boundary/initial value problem:

$$\text{Equation: } U_{xx} = U_x + U_t$$

$$\text{B.C.: } U=0 \text{ at } x=0$$

$$U=0 \text{ at } x=2$$

$$\text{I.C.: } U=f(x) \text{ when } t=0$$

$$U_t=g(x) \text{ when } t=0.$$

Here U_{xx}, U_t , and U_x are partial derivatives of U . If the solution is expressed as

$$U = \sum_{n=1}^{\infty} F_n(x) G_n(t).$$

please find out the expression of $G_n(t)$.

(25%)



4) Consider

$$f(z) = (z^2 - 1)^{-1}$$

where $z = x + iy$ is a complex variable. The Laurent expansion of $f(z)$ with $z = 1 + i$ as

the center can be expressed as $f(z) = \sum_{h=-\infty}^{\infty} C_h (z - 1 - i)^h$. If the region of convergence

of this expansion is $1 < |z - 1 - i| < \sqrt{5}$, please find out the coefficients C_2 and C_1 . (25%)