所別：工業管理研究所碩士班 甲組 科目：微積分

1. A sequence $\{x_n\}$ is said to be a Cauchy sequence if every $\varepsilon > 0$ there exists an integer N such that for all m and n satisfying $m \geq N$ and $n \geq N$, we have $|x_m - x_n| < \varepsilon$.

(a) (10 points) Show that every convergent sequence is a Cauchy sequence.
(b) (10 points) Show that every Cauchy sequence is bounded.

2. (15 points) A sequence of functions $\{f_n\}$ converges uniformly on E to a function f if for every $\varepsilon > 0$ there is an integer N such that $n \geq N$ implies $|f_n(x) - f(x)| < \varepsilon$.

Show that if a sequence of functions $\{f_n\}$, defined on E, converges uniformly on E to a function f, then there exists an integer N such that for all m and n satisfying $m \geq N$ and $n \geq N$, $x \in E$ implies $|f_n(x) - f_m(x)| < \varepsilon$.

3. (15 points) Let $G = \int_{t=a(x)}^{b(x)} f(t,x)\,dt$, and let X denote an interval subset of the real line. Suppose that $b(x) > a(x)$ for all $x \in X$ and both a and b are differentiable on X. Suppose further that f is continuous in the first argument and has a continuous partial derivative for $x \in X$ and $a(x) \leq t \leq b(x)$. Show

$$\frac{dG}{dx} = \int_{t=a(x)}^{b(x)} \frac{\partial f(t,x)}{\partial x}\,dt + f(b(x),x)\,b'(x) - f(a(x),x)\,a'(x).$$

注意：背面有試題

4. (20 points) The gamma function denoted by $\Gamma(n)$ is defined by

$$\Gamma(n)=\int_0^\infty x^{n-1}e^{-x}\,dx$$

which is convergent for $n>0$ and $\Gamma(n+1)=n\Gamma(n)$. Evaluate each of the following:

(a) $\dfrac{\Gamma(3)\Gamma(2.5)}{\Gamma(5.5)}$, (b) $\displaystyle\int_0^\infty x^6e^{-2x}\,dx$.

5. (10 points) Find the interval of convergence for $\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{n!}x^n$.

6. (10 points) Find the maximum and minimum values of $f(x)=32x-4x^2-12$ subject to the constraint conditions: $2x-30\le 0$ and $0\le x\le 6$.

7. (10 points) Find the characteristic polynomial, the eigenvalues, and the eigenvectors of $A=\begin{bmatrix}3 & -4\\ -5 & 2\end{bmatrix}$.