

1. Let  $X$  be exponential distributed with  $1/\lambda$ ; that is,  

$$f_X(x) = \lambda \exp(-\lambda x), 0 < x < \infty.$$

Find  $E(X|X > 1)$ . (20%)
2. The regression  $Y = \beta_0 + \beta_1 X + \varepsilon$  can be written in the standardized form as  $Z_Y = \alpha_1 Z_X + \alpha_2 Z_\varepsilon$ , where  $Z_W = (W - \mu_W)/\sigma_W$ ;  
  - (a) Write down  $\alpha_1$  and  $\alpha_2$  in terms of  $\beta_0$  and  $\beta_1$ . (10%)
  - (b) Show that  $\alpha_1^2 = 1 - \alpha_2^2$ . (10%)
3. Suppose that we have a random sample  $X_1, X_2, \dots, X_n$  from the normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Derive the  $100(1-\alpha)\%$  prediction interval for  $X_{n+1}$ . (20%)
4. A consumer product-testing organization wished to compare the annual power consumption for the five different brands of dehumidifier. Because power consumption depends on the prevailing humidity level, it was decided to monitor each brand at four different levels ranging from moderate to heavy humidity. Within each level, brands were randomly assigned to the five selected locations.  
  - (a) Is this a completely randomized experimental design? Explain your reason. (10%)
  - (b) Write down the appropriate model for the mean response and assumptions. (10%)
5. In two-sample t test, the numerators of the paired t and pooled t statistics are identical, but the difference between two test statistics is due entirely to the denominators. Suppose that there is positive dependence within pairs, compare the denominators between the paired t statistic and the pooled t statistics. (20%)