

所別：企業管理學系碩士班 一般類組(甲組) 科目：微積分

甲、填充題：共 4 大題，每題 10 分，共 40 分。請將答案依題號寫在答案卷上，不必寫演算過程。

1. Evaluate  $\int_C (x + y) ds = \underline{\hspace{2cm}}$ , where  $C$  is the straight-line segment  $x = t, y = (1 - t), z = 0$  from  $(0, 1, 0)$  to  $(1, 0, 0)$ .
2. The area of the region that lie inside the cardioid curve  $r = 1 + \cos \theta$  and outside the circle  $r = \cos \theta$  is  $\underline{\hspace{2cm}}$ .
3. Estimate how much  $\underline{\hspace{2cm}}$  the value of  $f(x, y, z) = y \sin x + 2yz$  will change if the point  $P(x, y, z)$  move 0.1 units from  $P_0(0, 1, 0)$  straight toward  $P_1(2, 2, -2)$ .
4. Find the circulation  $\underline{\hspace{2cm}}$  of the vector field  $\mathbf{F}(t) = (x - y)\mathbf{i} + x\mathbf{j}$  around the  $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$  for  $0 \leq t < 2\pi$ .

乙、計算題：共 6 大題，每大題 10 分，共 60 分。須詳細寫出演算過程、否則不予計分。

1. Evaluate the following integrals

$$(a) \int_0^3 a^x dx \quad a > 0, a \neq 1 \quad (2 \text{ 分}) \quad (b) \int_0^1 \frac{x^3 - 1}{\ln x} dx \quad (8 \text{ 分})$$

2. Find an equation for the tangent line that is (a) tangent and (b) normal to the curve  $\mathbf{r}(t)$  at the point determined by the given value of  $t$ .

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (t^2 - \cos t)\mathbf{j}, \quad t = 0$$

3. (a) Find  $\lim_{x \rightarrow 0^+} x \int_x^1 \frac{\cos t}{t^2} dt$ . (b) If  $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3$ , find  $\lim_{x \rightarrow 2} f(x)$ .

4. Evaluate the integrals.

$$(a) \int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx. \quad (b) \int \cos(\ln x) dx.$$

5. Let

$$a_n = \begin{cases} n/2^n, & \text{if } n \text{ is a prime number} \\ 1/2^n, & \text{otherwise} \end{cases}$$

Does  $\sum a_n$  converge? Give reasons for your answer.

6. Find all extrema of

$$f(x, y) = e^{-xy}$$

subject to the constraint  $x^2 + 4y^2 = 1$