

所別：企業管理學系碩士班 一般類組(乙組) 科目：工程數學

1. (10%) Solve the initial value problem:

$$y' = y + xy^2, y(0) = 1/2$$

2. (10%) Find the general solutions to the following differential equation.

$$y'' + 2y' + 2 = 0$$

3. (10%) Find the Laplace transform of the given function:

$$\frac{\sin kt}{t}$$

4. (10%) Compute the inverse of the given Laplace transform.

$$\frac{e^{-\pi s}}{1 + s^2}$$

5. (15%) Expand the function  $f(x) = x \sin x$ ,  $0 < x < 2\pi$ , in a Fourier series.

6. (10%) Find the LU decomposition of the given matrix.

$$\begin{bmatrix} 2 & -2 & 2 & 4 \\ -2 & 4 & 2 & -1 \\ 6 & -2 & 4 & 14 \end{bmatrix}$$

7. (15%) Find the least square solution to the following equation.

$$\begin{bmatrix} -9 & -9 & 4 \\ 5 & 4 & 9 \\ 5 & 7 & 7 \\ 6 & 2 & -5 \\ -7 & 4 & -4 \end{bmatrix} x = \begin{bmatrix} 6 \\ -4 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$

8. (20%) Consider a  $n \times n$  tridiagonal matrix,  $M_n$ , with 2 on the three central diagonals (other entries are zeros). e.g.,

$$M_6 = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

Let  $D_n$  denote the determinant of the tridiagonal matrix,  $M_n$ .

(a) Compute  $D_1, D_2$ . (4%)

(b) Show that  $D_n = 2 * D_{n-1} - 4 * D_{n-2}$ . (6%)

(c) Find  $D_{12}$  (Hint: Use eigenvalues). (10%)