

# 國立中央大學八十四學年度碩士班研究生入學試題卷

所別：財務管理研究所

組 科目：微積分

共 | 頁 第 | 頁

## 注意事項：

- 請列出計算過程，僅有答案，不予計分。
- 請務必將題號標示清楚，可不按順序作答。

1. (12%) Evaluate  $\lim_{n \rightarrow \infty} \sum_{k=1}^{[\sqrt{3n}]} \frac{k}{n^4} \sqrt{9n^4 - k^4}$ .

2. (12%) Let  $f(x)$  be the function given by

$$f(x) = \begin{cases} x^{-1} \sin(6x^2), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

- (a) Is  $f(x)$  continuous at  $x = 0$ ? Why?  
 (b) Is  $f(x)$  differentiable at  $x = 0$ ? Why?

3. (14%) Evaluate the following items.

(a)  $\int_0^2 \int_0^{\sqrt{3x}} \sqrt{x^2 + y^2} dy dx + \int_2^4 \int_0^{\sqrt{16-x^2}} \sqrt{x^2 + y^2} dy dx$ .

(b)  $\iint_R 3(x+y)^2 \sin(x-y) dA$ , where  $R$  is the region bounded by the rectangle with vertices  $(0, 0)$ ,  $(1, 1)$ ,  $(0, 2)$ , and  $(-1, 1)$ .

4. (14%) Let  $f(x) = \ln(x-1)$ .

- (a) Use the 2nd order Taylor polynomial  $p_2(x)$  to estimate  $\int_2^{2.1} f(x) dx$ .  
 (b) Without using calculator, estimate the error between the approximation obtained in (a) and  $\int_2^{2.1} f(x) dx$ .

5. (28%) Determine the convergence or divergence of the following series.

(a)  $\sum_{n=1}^{\infty} \frac{1 - \cos \frac{1}{n}}{n}$

(b)  $\sum_{n=1}^{\infty} \frac{(\ln n)^4}{n^2}$

(c)  $\sum_{n=2}^{\infty} \frac{1}{n^{1/2} (\ln n)^3}$

(d)  $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^{3/2}}$

6. (20%) Let  $f(x) = 1/(2+x^3)$ .

- (a) Find the power series centered at  $x = 0$  for  $f(x)$  and determine its interval of convergence.  
 (b) Let  $h(x) = \ln(2+x^3)$ . Find the power series centered at  $x = 0$  for  $h(x)$ , and also determine its interval of convergence.  
 (c) Evaluate  $h^{(33)}(0)$  and  $h^{(50)}(0)$ .

