The First Part: Statistics (50%)

1. Sample variances of x are usually calculated by dividing the sum of x minus xbar squared by N-1 which "correct for degrees of freedom." What does this correction accomplish? (5%)

2. For a fee of $2 you toss three fair coins and are paid (x^2-x) where x is the number of heads thrown. What is your expected profit from playing? (7%)

3. Suppose x is distributed uniformly between a and b. Derive the formula for E(x) and V(x), in terms of a and b. (8%)

4. Suppose you have T observations from the density $f(x)=\lambda x^{\lambda-1}$ for $0 \le x \le 1$. Find a method of moments estimator for λ. (10%)

5. Suppose x and y are random variables taking on values of zero or one, with probability distributed defined by

$$p(x=1)=\alpha$$
$$P(y=1|x)=e^{\beta x}/(1+e^{\beta x})$$

   (1) Given the random sample of size N on (y, x), find the maximum likelihood estimate of α and β. (13%)
   (2) Suppose that in your sample of observations with x = 1, half have y = 1. What is your estimated $\text{prob}(y=0|x=1)$? (7%)

Second Part: Basic Econometrics (50%)

Answer whether the following statements are true, false, or uncertain with reasons. No reasons, no points.

6. Suppose the classical linear regression model applies to $y=\beta x+\varepsilon$. The slope coefficient in the regression of x on y is just the inverse of the slope from the regression of y on x. True, false, or uncertain? Explain. (10%)

7. The estimator of the variance-covariance matrix of the OLS estimator becomes smaller when a relevant explanatory variable is omitted. True, false or uncertain? Explain. (10%)

8. Consider applying OLS to a consumption function $C = \alpha + \beta Y$ and to the corresponding saving function $S = \gamma + \delta Y$ where for all observations $Y = C + S$. The sum of squared residuals is the same for each regression. True, false, or uncertain? Explain. (10%)

9. Imposing a linear constraint on a regression will raise R^2 if the constraint is true and lower R^2 if it is false. True, false, or uncertain. Explain. (10%)

10. In a classical linear regression model, multicollinearity leads to bias, not in the estimation of the regression coefficients themselves, but rather in the estimation of their variances. True, false, or uncertain? Explain. (10%)