

系所別: 資訊管理學系 丙組 科目: 離散數學

1. An RNA chain is a string on the alphabet $\{A, C, G, U\}$.
 - (a) Find the probability of a length 6 RNA chain consisting of 3 C's and 3 A's. (5%)
 - (b) Count the number of length k RNA chains with even number of U's. (15%)
2. Solve the following recurrence relation. (10%)

$$a_{n+3} = 9a_{n+2} - 24a_{n+1} + 20a_n, n \geq 0; a_0 = 1, a_1 = 2, a_2 = 0.$$

3. For $n \geq 3$, count the number of different Hamilton cycles in each of the following graphs.
 - (a) K_n (the complete graph with n vertices) (10%)
 - (b) W_n (the wheel graph consisting of a cycle with n vertices together with an additional vertex that is adjacent to the n vertices of the cycle) (10%)

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4. Let $H_k = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}$ for $k \geq 1$. Prove that $H_{2^n} \geq 1 + \frac{n}{2}$ for $n \geq 0$. (10%)
5. Let U be a universal set and $X \subseteq U$. Define the characteristic function of X (in U) as follows:

$$C_X(u) = \begin{cases} 1 & \text{if } u \in X \\ 0 & \text{if } u \notin X \end{cases}$$

Prove that $C_{X \cup Y}(u) = C_X(u) + C_Y(u)$ for all $u \in U$, if and only if $X \cap Y = \emptyset$. (15%)

6. Let the input be always a bit string, please draw the transition diagram of a finite-state machine which outputs 1 if k 1's have been input, where k is a multiple of 3, and outputs 0 otherwise. (10%)
7. Let n be the product of two prime numbers, $\phi(n)$ be the number of elements in the reduced set of residues module n , and integers e, d satisfy the following equation:

$$ed \bmod \phi(n) = 1.$$

Now given a message $M \in [0, n-1]$, and $\gcd(M, n) = 1$, prove that $(M^e \bmod n)^d = M^{ed} \bmod n$. (15%)