國立中央大學九十一學年度碩士班研究生入學試題卷

1. An AM modulator operates with the message signal

$$m(t) = 10\cos(40\pi t) + 20\cos(80\pi t)$$

The unmodulated carrier is given by $100\cos(2*1000*10^3\pi t)$, and the system operates with an index of 1/2

(a) Write the equation for $m_n(t)$, the normalized signal with a minimum value of -1 (3 %)

(b) Determine $< m_n^2(t) >$, the power in $m_n(t)$. (4 %)

(c) Determine the efficiency of the modulator. (4 %)

(d) Roughly sketch the double-sided spectrum of $x_c(t)$ (amplitude and phase spectrum), the modulator output for all frequency components. (4 %)

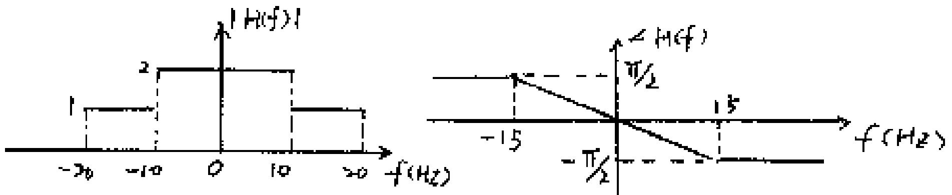
2. Assume the system is linear time-invariant (LTI).

(a) The impulse response $h(t)$ of the LTI system is $h(t)$. Let the input to the LTI system equal to

$$x(t) = \sum_{n=1}^{N} \alpha_n \delta(t - t_n)$$

Please find the output $y(t)$ in terms of $\alpha_n, h(t)$, and t_n. (5 %)

(b) Assume we have the system with the amplitude response $|H(f)|$ and the phase response $\angle H(f)$ listed below. (i.e. $\mathcal{F}\left\{h(t)\right\} = H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$, where $\mathcal{F}\left\{\right\}$ is the Fourier transform.) Determine the outputs $y_1(t)$ whether $y_1(t)$



"distortionless" with the input signal $x_1(t)$. (i.e. $y_1(t) = \mathcal{H}\left\{x_1(t)\right\}$) where $x_1(t) = A_1 e^{j2\pi 4t} + A_2 e^{j2\pi 8t} + A_3 e^{j2\pi 12t}$. Please also calculate the output $y_1(t)$. (5 %)

(c) In wireless communication systems, the multipath scenario is quite common. Denote $s(t)$ the transmitted signal and $z(t)$ the received signal. If we have three multipath components, and then we have

$$z(t) = \beta_1 s(t - \tau_1) + \beta_2 s(t - \tau_2) + \beta_3 s(t - \tau_3)$$

, where β_i is the channel gain of the i-th multipath component and τ_i is the i-th multipath propagation delay, $i = 1, \cdots, 3$. Assume the wireless channel varies very slowly and we would like to model the wireless channel as the LTI system in some time period. The multipath wireless channel is described by the impulse response $h_s(t)$ by viewing $s(t)$ and $z(t)$ as the input and output signals. Please find the mathematical expression $h_s(t)$ in terms of β_i and τ_i. (5 %)

參考用

注意：背面有試題

3. Let a random process be given as

$$Z(t) = X(t)\cos(\omega_0 t + \theta)$$

where $X(t)$ is a stationary random processes with $E\{X(t)\} = 0$, $E\{X^2(t)\} = \sigma_X^2$ and $E\{X(t)X(t+\tau)\} = R_x(\tau)$.

(a) If $\theta = 0$, find $E\{Z(t)\}$ and $E\{Z^2(t)\}$. Is $Z(t)$ stationary? Provide your explanation. (5 %)

(b) If θ is a random variable independent of $X(t)$ and uniformly distributed in the interval $(-\pi, \pi)$, show that $E\{Z(t)\} = 0$ and $E\{Z^2(t)\} = \sigma_X^2/2$. Is $Z(t)$ wide-sense stationary? Provide your explanation. (5 %)

(c) Let

$$Z(t) = X(t)\cos(\omega_0 t + \theta) + Y(t)\sin(\omega_0 t + \theta)$$

where $X(t)$ and $Y(t)$ are stationary Gaussian random processes with $E\{X(t)\} = E\{Y(t)\} = 0$, $E\{X^2(t)\} = E\{Y^2(t)\} = \sigma^2$, and $E\{X(t)X(t+\tau)\} = E\{Y(t)Y(t+\tau)\} = R(\tau)$. $X(t)$ and $Y(t)$ are uncorrelated for any t. If θ is a random variable independent of $X(t), Y(t)$ and uniformly distributed in the interval $(-\pi, \pi)$, Find $E\{Z(t)\}$ and $E\{Z^2(t)\}$. Is $Z(t)$ stationary? Provide your explanation. (10 %)

4. Consider an optimal coherent detector for QPSK. Assume that QPSK signals are transmitted over the AWGN channel with double-sided power spectral density $\frac{N_0}{2}$. If there is a phase error θ, which is the difference between the unknown carrier phase and the phase estimate at the receiver. Assume that $0 \le \theta < \frac{\pi}{4}$. Compute the detected error probability in terms of $\frac{E_s}{N_0}$ and θ, where E_s denotes the energy per symbol. (20%)

5. Consider 16APSK with signals given as

$$s_i(t) = A_i cos(\omega_c t + \frac{\pi(B_i - 1)}{4}), \qquad 0 \le t \le T_s,$$

where $A_i \in \{a, 2a\}$ with equal probability as well as $b_i \in \{1, 2, \cdots, 8\}$ with equal probability. Assume that the signals are transmitted over the AWGN channel with double-sided power spectral density $\frac{N_0}{2}$.

(a) Represent a in terms of E_s and T_s, where E_s denotes the energy per symbol and T_s denotes the signalling interval. (5%)

(b) Devise an optimal coherent detector and show the optimal decision regions . (10%)

6. Consider a communication system as follows. At the transmitter, one information bit is fed into a channel encoder to obtain a 3-bit output. Assume that the channel code used herein is a repetition code. Then the three coded bits are sent into a BPSK modulator successively. Suppose that BPSK signals are transmitted over the AWGN channel with double-sided power spectral density $\frac{N_0}{2}$. At the receiver, a matched filter is used to produce the detected value of coded bits successively. Then the decoder makes a decision of the information bit according to the three bits. Represent the error probability of the information bit in terms of $\frac{E_b}{N_0}$, where E_b denotes the energy per information bit. (15%)