所別：通訊工程學系碩士班 丙組 科目：工程數學

注意：本考題分為機率、線代與離散 數學三部份，考生任選兩部份作答(但所選的部份各小題必須全部作答)，總分為 100 分。

PART Ⅰ：機率

1. A resistor may come from any one of the three manufacturers A, B, and C with probabilities $P_A = 0.25$, $P_B = 0.50$, and $P_C = 0.25$. The probabilities that the resistor will be defective equal 0.01, 0.02, and 0.04, respectively.

(10%) (A) Compute the probability that a randomly chosen resistor will be defective.

(10%) (B) If the chosen resistor is defective, what is the probability that this resistor comes from manufacture B?

2. (10%) Let X be a computer-generated random variable which is uniformly distributed in (0, 1).

Find the distribution of the random variable Y which is defined by：

$Y = -\frac{1}{\lambda}\ln(1-X)$ ， $\lambda > 0$.

3. The score of each student is rounded off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5).

(10%) (A) Find the expectation and variance of the rounding error.

Assume a class consists of 100 students. The 100 scores are averaged to take the average score and associated error is called the average rounding error.

(10%) (B) What is the expectation and variance of the average rounding error?

參考用

注意：背面有試題

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PART II：線性代數

1. (10 %) Consider the system of linear equations $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & k+1 & 6 \\ -1 & 3 & k-2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 8 \\ -1 \end{bmatrix}$$

Determine the values of k such that:

(a) The system has infinitely many solutions.

(b) The system has a unique solution.

(c) The system has no solution.

2. (10 %) Let T : $\mathbf{R}^{2\times 2} \to \mathbf{R}^{2\times 2}$ be the linear transformation given by

$$\mathbf{T}(\mathbf{A}) = \mathbf{A}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{A}$$

Find a basis for the kernel (nullspace) and a basis for the image of **T**.

3. (10 %) Let **A** be the matrix $\mathbf{A} = \begin{bmatrix} -2 & 4 \\ 1 & 1 \end{bmatrix}$

(a) Find an invertible matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$.

(b) Compute $\mathbf{A}^{100}$.

(c) Find a square matrix **B** such that $\mathbf{B}^5 = \mathbf{A}$.

4. Give the correct choice of the following statements. (單選)

(a) (5 %) Let **V** and **W** be subspace of R^5 such that dim(**V**) = 4 and dim(**W**) = 2. Then the possible dimension for $\mathbf{V} \cap \mathbf{W}$ is

(i) 0,1 (ii) 1,2 (iii) 2,3 (iv) none of above

(b) (5 %) If $\{u_1, \ldots, u_m\} \subset R^4$ is linear independent and $\{v_1, \cdots, v_l\}$ spans R^4, then

(i) $l, m \geq 4$ (ii) $l, m \leq 4$ (iii) $l \geq 4, m \leq 4$ (iv) $l \leq 4, m \geq 4$ (v) none of the above

(c) (5 %) If 3 is an eigenvalue of **A**, then $\mathbf{A}^2 - 4\mathbf{A} + 3\mathbf{I}$ is invertible (i) True (ii) False (答錯倒扣 5%)

(d) (5 %) The matrix $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ are similar. (i) True (ii) False (答錯倒扣 5%)

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PART Ⅲ：離散數學

1. [10 %] Determine a shortest path between a and z in the graph in Figure 1, where the numbers associated with the edges are the distances between vertices. Please show each step in determining the shortest path.

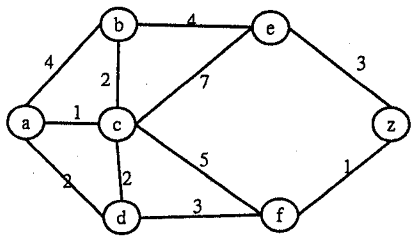

Figure 1

2. [14 %]

Find a minimum cost spanning tree of the graph in Figure 2. Please also describe the algorithm in determining the minimum cost spanning tree.

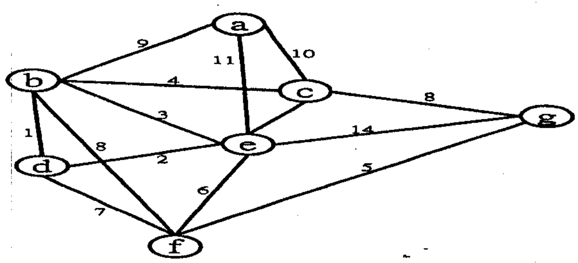

Figure 2

3. [12 %] Solve the recurrence relation: $a_n = 3a_{n-1} - 2, a_0 = 2$ by using

(a) characteristic polynomial (b) generating function.

4. [14 %]

(a) Find a finite state machine that recognizes the set of strings of 0s and 1s in each of which there is an even number of 1s.

(b) Given a description (verbal or in set-theoretic notation) of the set of strings recognized by the finite state machine in Figure 3.

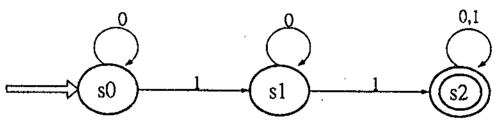

Figure 3

參考用