1. (10%) Find the most general analytic function $f(z) = u(x,y) + iv(x,y)$ such that $u(x,y) = x^2 - x - y^2$.

2. (12%, 3% each) Integrate

$$f(z) = \frac{z^2 + 1}{z^2 - 1}$$

in the counterclockwise sense around a circle of radius 1 with center at the point

(a) $z = 1$,　(b) $z = \frac{1}{2}$,　(c) $z = -1 + i\frac{1}{2}$,　(d) $z = i$

3. (8%, 4% each) Consider tossing a fair coin independently ten times. (Note that a coin has head on one side and tail on the other side.)

   (a) Find the probability of the event that there are six heads in any order.

   (b) Find the probability of the event that there are less than five heads in the first eight tosses.

4. (10%) Please diagonalize the matrix $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ -4 & 0 & 0 \end{bmatrix}$.

5. (10%) Please evaluate $\oint_c (x^3 + 3y)dx + (2x - e^{y^2})dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

6. (15%) Please find the solution of $X' = \begin{bmatrix} -3 & 1 \\ 2 & -4 \end{bmatrix}X + \begin{bmatrix} 3t \\ e^{-t} \end{bmatrix}$.

7. (15%, 5% each) Please solve the following differential equations
   (a) $y' = 1/(6e^y - 2x)$;　(b) $xy'' - y' = (3+x)x^2e^x$;
   (c) $y'' + 3y' + 2y = \delta(t-3)$, $y(0) = 0$, $y'(0) = 0$, where $\delta(t)$ is a unit impulse function.

8. (10%, 5% each)
(a) Please find the Laplace transforms
   $f(t) = t$, if $0 < t < 1$; $f(t) = 1$, if $1 < t < 2$; $f(t) = 0$, otherwise.,
(b) Please find the Inverse Laplace transform $L^{-1}(F(s))$ and plot the result
   $f(t)$, where $F(s) = \frac{k}{ps^2} - \frac{ke^{-ps}}{s(1-e^{-ps})}$ and both p and k are constants.

(9) (10%) Determine the charge q(t) (on the capacitor) and current i(t) for a series circuit in which L=1 henry, R=20 ohms, C=0.01 farad, E(t)=120sin 10t volts, q(0)=0, and i(0)=0. Moreover, what is the steady state current?