國立中央大學八十四學年度碩士班研究生入學試題卷

所別：資訊工程研究所　　組　科目：線性代數　　　　共 / 頁 第 / 頁

※ 請務必按照題號次序做答。

1. (50%) True or False. （一定要有說明、證明或反例。每小題 5 分）
   (a) Every matrix is row equivalent to a unique matrix in echelon form.
   (b) For the linear system $\mathbf{A}_{m\times n}\,\mathbf{x}_{n\times 1} = \mathbf{b}_{m\times 1}$. **A** has infinitely many solution if and only if at least one column of **A** doesn't contain a pivot position.
   (c) The linear system $\mathbf{Ax} = \mathbf{b}$ with more equations than variables cannot have a unique solution.
   (d) If the columns of **A** are linearly independent, then the linear system $\mathbf{Ax} = \mathbf{b}$ has solution.
   (e) If matrices $\mathbf{AB} = \mathbf{AC}$, then $\mathbf{B} = \mathbf{C}$.
   (f) If matrices **A** and **B** are row equivalent then their column spaces are the same, but their row spaces may be different.
   (g) If matrix $\mathbf{A}_{n\times n}$ has n independent eigenvectors, then **A** has n distinct eigenvalues.
   (h) If both $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ and $\{\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4\}$ are linearly independent sets, then $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4\}$ is linearly independent, where vectors $\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3$, and $\mathbf{v}_4$ are in $\boldsymbol{R}^4$.
   (i) If V is orthogonal to W, then $V^\perp$ is orthogonal to $W^\perp$, where $V^\perp$ is the orthogonal complement of V.
   (j) $V \cap V^\perp$ may be an empty set.

2. (10%) Give four methods to determine a linear system $\mathbf{A}_{m\times n}\,\mathbf{x}_{n\times 1} = \mathbf{b}_{m\times 1}$ has solution.

3. (10%) **A** and **B** are square matrices. Prove that if either $\mathbf{BA} = \mathbf{I}$ or $\mathbf{AB} = \mathbf{I}$, then **A** and **B** are invertible, with $\mathbf{B} = \mathbf{A}^{-1}$ and $\mathbf{A} = \mathbf{B}^{-1}$.

4. (10%) The inverse of block matrix $\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{A} & \mathbf{I} & \mathbf{0} \\ \mathbf{B} & \mathbf{C} & \mathbf{I} \end{bmatrix}$ is $\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{X} & \mathbf{I} & \mathbf{0} \\ \mathbf{Y} & \mathbf{Z} & \mathbf{I} \end{bmatrix}$. Find matrices **X**, **Y**, and **Z**.

5. (10%) Find $\mathbf{A}^{33}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, where $\mathbf{A} = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$.

6. (10%) Find a QR factorization of matrix $\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \end{bmatrix}$.