系所別：資訊工程學系，網路學習科技研究所　　科目：離散數學

1. (20%) Calculate the following summations where n, r, an s are positive integers

(a) $\sum_{k=0}^{\infty} C(n, k)$

(b) $\sum_{k=0}^{\infty} (-1)^k C(n, k)$

(c) $\sum_{k=0}^{\infty} k\, C(n, k)$

(d) $\sum_{k=0}^{\infty} C(n, k)^2$

(e) $\sum_{k=0}^{n} C(k, r)$

(f) $\sum_{j=0}^{r} C(n-j, r-j)$

(g) $\sum_{k=0}^{\infty} C(r, k)C(s, n-k)$

2. (20%) Find the logical equivalence relations among the following compound statements

(a) $\neg p \vee q \vee r$

(b) $p \vee \neg q \vee r$

(c) $p \vee q \vee \neg r$

(d) $(p \to q) \to r$

(e) $p \to (q \to r)$

(f) $\neg p \vee q \vee \neg r$

(g) $p \wedge q \vee \neg r$

(h) $(p \wedge q) \to r$

(i) $p \vee \neg q \vee \neg r$

(j) $\neg p \vee \neg q \vee r$

參考用

3. (a) (10%) Find the smallest positive integer g such that $204m + 330n = g$ for some integers m and n.

(b) (10%) Find the inverse of 34(mod 89); that is an integer a such that $a \times 34 \pmod{89}$ is equal to 1. Please do not just give the answer and clearly describe your process to get the answer.

4. (a) (10%) Find a recurrence relation for a_n, the number of ways a sequence of 1's and 3's can sum to n. For example, $a_4 = 3$ since 4 can be obtained with the following sequences: 1111 or 13 or 31. Let α be the real number such that $\lim_{n\to\infty} a_n / \alpha^n = 1$. Estimate the value of α with 1 digit to the right of the decimal point.

(b) (10%) Find and solve a recurrence relation for the number of n-digit quinary sequences that have an even number of 0's (quinary sequences using only the digits 0, 1, 2, 3, 4). Use your result to compute the number when $n = 100$.

注意：背面有試題

5. (a) (10%) A directed graph is called a ***directed complete simple graph*** if for every two vertices $\boldsymbol{u}$ and $\boldsymbol{v}$ in the graph there exists one and only one directed edge from $\boldsymbol{u}$ to $\boldsymbol{v}$ or from $\boldsymbol{v}$ to $\boldsymbol{u}$. Prove by induction: for all integers $\boldsymbol{n} \geq 2$, the directed complete simple graph on $\boldsymbol{n}$ vertices has a directed path that passes through every vertex of the graph exactly once.

(b) (10%) A graph G is said to be ***planar*** if it can be drawn on a plane without any crossovers and the drawing divides the plane into regions. Intuitively, the regions are the connected portions of the plane remaining after all the curves and points of the plane corresponding, respectively, to edges and vertices of G have been deleted. The vertices and edges incident with a region make up the boundary of the region. If G is connected, then the boundary of a region is a closed path in which each cut-edge of G is traversed twice, and each non-cut-edge is traversed once. When the boundary contains no cut edges of G, then the boundary is a cycle of G. The degree of a region is the length of its boundary. **Fill in the blanks** for the following three paragraphs about graphs and planar graphs:

Suppose that a graph G has 2 vertices of degree 4, 4 vertices of degree 3, and 2 vertices of degree 5. Then G has ______ edges.

If a planar connected graph G has 5 regions of degree 4, 11 regions of degree 5, 2 regions of degree 7, 2 regions of degree 8, and 1 regions of degree 9, then G has ______ edges and ______ vertices.

If a planar graph G has 15 vertices, 19 edges, and 8 regions, then G has ________ connected components.