國立中央大學九十學年度碩士班研究生入學試題卷

所別：資訊工程學系　不分組　科目：離散數學　共 1 頁　第 1 頁

答題說明：總共有 5 題，每題 20 分，若為計算題請務必將求解過程寫出。

1. (a) What is Principle of Inclusion and Exclusion (PIE)?
   (b) Apply PIE to calculate the number of primes that are no larger than 110.

2. Let D_n be the number of derangements of 1, 2, ..., n. Use combinatorial argument to show that $D_n = (n-1)(D_{n-1} + D_{n-2})$ for $n \geq 3$.

3. In a room where there are more than 50 people with ages between 1 and 100, show that
   (a) Either two people have the same age or there are people whose ages are consecutive integers.
   (b) Either two people have the same age or one person's age is a multiple of another's.
   (c) Some of the people shake hands. Show that at least two shook the same number of hands. ("No hands" is a possibility.)

4. Let $G=(V, E)$ be a simple graph (i.e. G contains no multiple edges or self-loops). For each vertex v in V, let deg(v) denote the degree of v (i.e. the number of edges incident to v). A cycle in G is a sequence of distinct vertices: $v_1, v_2, ..., v_k$, where $k \geq 3$, $(v_k, v_1) \in E$ and $(v_i, v_{i+1}) \in E$ for $1 \leq i \leq k-1$, and we say that k is the length of the cycle. Show that:
   (a) The number of odd degree vertices must be even.
   (b) If $\deg(v) \geq 2$, for every vertex v in V, then G must contain a cycle.
   (c) If $\deg(v) \geq d$, for every vertex v in V and some integer $d \geq 2$, then G must contain a cycle of length $d+1$.

5. Let $a_n = 1^3 + 2^3 + \ldots + n^3$. Evaluate a_n (i.e. find a formula for the sum) by using the following two methods:
   (a) Derive a recurrence relation for a_n and then solve the relation by the standard method of solving linear recurrence relations with constant coefficients.
   (b) Derive a generating function for a_n and then use this generation function to evaluate the sum.