所別：電機工程學系碩士班 乙組(一般生) 科目：近代物理

(學位在職生)

(1) Show that the energy density U(T) of a blackbody radiation field in equilibrium at the temperature T is directly proportional to T^4. (10%)

(2) If a particle is in the state (limited in one dimensional system)

$\psi(x,t) = \frac{1}{\sqrt{a\sqrt{2\pi}}} \exp\left[\frac{-(x-x_0)^2}{4a^2}\right] \exp(\frac{ip_0 x}{\hbar}) \exp(-i\omega_0 t)$, calculate its expectation value of momentum. (15%)

(3) Consider the time-dependent Schrodinger equation

$\frac{\partial}{\partial t}\psi(r,t) + \frac{i\hat{H}}{\hbar}\psi(r,t) = 0$, where $\hat{H}$ is a time-independent Hamiltonian. Show that

$\psi(r,t) = \exp(\frac{-it\hat{H}}{\hbar})\psi(r,0)$.(20%)

(4) Based on time-dependent Schrodinger equation

$\frac{\partial}{\partial t}\psi(r,t) + \frac{i\hat{H}}{\hbar}\psi(r,t) = 0$, derive the continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$, where ρ and J denote, respectively, the number density and current density. (15%)

(5) For the Hamiltonian of a harmonic oscillator $\hat{H} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{m\omega_0^2}{2}x^2$, we introduce the annihilation and creation operators defined as $\hat{a} = \frac{\sqrt{m\omega_0}}{\sqrt{2\hbar}}(\hat{x} + \frac{i\hat{p}}{m\omega_0})$ and $\hat{a}^+ = \frac{\sqrt{m\omega_0}}{\sqrt{2\hbar}}(\hat{x} - \frac{i\hat{p}}{m\omega_0})$, respectively. Show that the Hamiltonian can be rewritten as $\hat{H} = \hbar\omega_0(\hat{a}^+ a + \frac{1}{2})$.(10%)

(6) Consider a particle of mass M confined to the interior of a rectangular box with impenetrable walls, (edge lengths L_1, L_2, L_3). Find the eigenfunctions and eigenvalues for this rectangular box. (15%)

(7) Write down the Hamiltonian of an electron in a uniform, constant magnetic field B which points in the z direction. (15%)