類組:B-7

1 (20%) Find the general solutions or the particular solutions, which satisfy the given initial conditions, for the following differential equations:

- (a) y'' + 3.2y' + 2.56y = 0.
- **(b)** 2y'' 9y' = 0.
- (c) y'' + 0.4y' + 0.29y = 0, y(0) = 1, y'(0) = -1.2.
- (d) $y'' k^2 y = 0$ $(k \neq 0)$, y(0) = 1, y'(0) = 1.

2 (10%) It is obvious that e^x satisfies the differential equation

$$y'' - y = 0.$$

Then, by using the method of REDUCTION OF ORDER, please prove that e^{-x} is another basis for this equation.

3 (10%) Solve y' = -2xy by the method of power series.

4 (15%) If f(t) has the Laplace transform of F(s) and the operator $\mathcal{I}\{$ $\}$ denotes the Laplace transformation, please prove the followings:

- (a) $\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}F(s)$, here u is the unit step function.
- **(b)** $\mathcal{I}\{\delta(t-a)\}=e^{-as}$, here δ is the Dirac's delta function.
- (c) $(\delta * f)(t) = f(t)$, here * means the convolution operator. (Hint: Use the convolution theorem.)

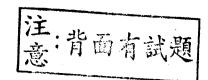
5 (10%) For an infinite bar one-dimensional heat equation $\frac{\partial u(x,t)}{\partial t} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2},$ with the initial condition u(x,0) = f(x), has an solution of the error function form

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x+2cz\sqrt{t})e^{-z^2}dz.$$

If f(x) = 1 when x > 0 and f(x) = 0 when x < 0, please show that

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\frac{x}{2c\sqrt{t}}}^{\infty} e^{-z^2} dz \qquad \text{for } t > 0$$





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6 (15%) The matrices $P1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $P2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ and $P3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ occur in the quantum mechanical theory of electron spin and are called *Pauli spin matrices*, in honor of the physicist Wolfgang Pauli (1900-1958). Here *i* is the imaginary unit.

- (a) Verify that they all have eigenvalues 1 and -1.
- (b) Determine all 2x2 matrices with complex entries having the two eigenvalues 1 and -1.
- (c) Show that any rank 2 matrix M can be represented as a sum of a 2x2 unit matrix and these Puli spin matrices.

7(20%) A general form of the solution within the spatial interval (0,L) for the one-dimensional wave equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

with two boundary conditions:

$$u(0,t) = 0$$
 and $u(L,t) = 0$ for all t

and two initial conditions:

$$u(x,0) = f(x)$$
 and $\frac{\partial u}{\partial t}\Big|_{t=0} = g(x)$

is

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} (B_n \cos \frac{cn\pi}{L} t + B_n^* \sin \frac{cn\pi}{L} t) \sin \frac{n\pi}{L} x$$

where
$$B_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$
 and $B_n^* = \frac{2}{cn\pi} \int_0^L g(x) \sin \frac{n\pi x}{L} dx$.

Now please write the explicit solution, i.e. don't use the notation of Σ as the final solution and write down explicitly the non-zero u_n 's up to four terms, of the one-dimensional wave equation corresponding to the triangular initial deflection

$$f(x) = \begin{cases} 2kx/L & \text{if } 0 < x < L/2 \\ 2k(L-x)/L & \text{if } L/2 < x < L \end{cases}$$

and initial velocity zero. (*Hint*: The coefficient b_n of the Fourier Sine Series for the odd periodic extension of f(x) is $b_n = \frac{8k}{n^2\pi^2} \sin \frac{n\pi}{2}$.)

