一. 填充題(每題八分)

1. Suppose that $f(x)=\sin^{-1}(\ln(3x+2))$. Then $f'(x)=$ __甲__.

2. Let $p=(0,-1)$ be a point of the curve $C: x^2+xy+y^2-x=1$. The tangent line equation of C at p is __乙__.

3. Let $H(x)=\int_0^{x^2}\frac{1}{1+t^3}dt$ and $L(x)=\int_0^{x}\frac{1}{1+t^3}dt$. Then $H'(2)-L'(4)=$ __丙__.

4. $\int x\sec^2 x\,dx=$ __丁__.

5. Which of the following series is convergent? __戊__

(a) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+1}$

(b) $\sum_{n=1}^{\infty}\sin(\sqrt{n}+1)$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}+1}$

(d) $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}+1}\right)^n$

二. 計算與證明題(每題十二分)

1. Let $f:\mathbb{R}^2\to\mathbb{R}$ be differentiable. Given $\mu=(\frac{3}{5},\frac{4}{5})$, $\nu=(\frac{4}{5},-\frac{3}{5})$. Prove that $\|\nabla f\|^2=|f'_\mu|^2+|f'_\nu|^2$, where f'_μ and f'_ν are the directional derivative of f in the direction of μ and ν, respectively.

2. Find the absolute maximum and absolute minimum of $f(x, y, z) = xy + z^2$ on $x^2 + y^2 + z^2 \leq 1$.

3. Find the volume of the solid T that is bounded above by the cone $z^2 = x^2 + y^2$, below by the x-y plane and on the sides by the hemisphere $z = \sqrt{4 - x^2 - y^2}$.

4. Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{1 + a_n}$ converges.

5. Let $f(x)$ be a differentiable function. Suppose that $f'(x)$ is continuous on $[a, b]$, $f'(a) > 0$ and $f(b) < f(a)$. Prove that there exists $c \in (a, b)$ such that $f'(c) = 0$.