# Examination on

## Linear Algebra

1. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$
   (a) Find $rank(A), rank(B)$, (5%)
   (b) Find the eigen values of $A$ and $B$.(5%)

2. Let $A = \begin{bmatrix} 2 & 8 & 2 \\ 8 & -4 & -10 \\ 2 & -10 & -7 \end{bmatrix}$.
   (a) Find an orthogonal matrix $S$ such that $S^T \cdot A \cdot S$ is a diagonal matrix, (15%)
   (b) Use eigen value technique to determine whether the curve defined by : $2x^2 - 4y^2 + 16xy + 4x - 20y - 7 = 0$ is an ellipse, hyperbola, parabola or a pair of straight lines.(10%)

3. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.
   (a) Find the projection vector $p$ of $b$ onto the column space of $A$, (10%)
   (b) Find all solutions of the matrix equation $A \cdot x = p$, (10%)
   (c) Find a solution $x_0$ satisfies :
      (I) $A \cdot x_0 = p$,
      (II) $|x_0| \leq |x|$ if $A \cdot x = p$.(10%)

4. Let $A$ be a real $m \times n$ matrix. Prove the mapping $x \longmapsto y = A \cdot x$ define an isomorphism (one to one and onto linear mapping) from the row space of $A$ onto the column space of $A$. (15%)

5. Prove or disprove the following statements: (20%)
   (a) $rank(A) = rank(A^T * A)$ for all real $m \times n$ matrix $A$,
   (b) $rank(A) = rank(A^T * A)$ for all complex $m \times n$ matrix $A$ ,
   (c) $rank(A) = rank(\bar{A}^T * A)$ for all complex $m \times n$ matrix $A$ , where $\bar{A}$ is the conjugate of $A$.