國立中央大學八十七學年度轉學生入學試題卷

數學系　三年級　　　科目：微分方程　　　

(10分) 1. Solve the differential equation

$$(y\cos x + 2xe^{y}) + (\sin x + x^2e^{y} - 1)y' = 0.$$

(10分) 2. Find the solution of

$$y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8e^{t}\cos 2t.$$

(10分) 3. Given that $y_1(t) = e^{t}$, $y_2(t) = te^{t}$, and $y_3(t) = e^{-t}$ are solutions of the homogeneous equation corresponding to

$$y''' - y'' - y' + y = g(t). \qquad (*)$$

Use the method of variation of parameters to determine a particular solution of (*) in terms of an integral.

(10分) 4. Find a series solution in powers of the differential equation

$$y'' - xy = 0, \qquad -\infty < x < \infty.$$

(10分) 5. Find the solution of the initial value problem

$$2y'' + y' + 2y = \delta(t-5)$$

$$y(0) = 0, \quad y'(0) = 0,$$

where the function δ is defined to have the properties

$$\delta(t-5) = 0, \quad t \neq 5;$$

$$\int_{-\infty}^{\infty} \delta(t-5)\,dt = 1.$$

(10分) 6. Find the general solution of the system

$$X' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} X + \begin{pmatrix} 2e^{t} \\ 3t \end{pmatrix}$$

(10分) 7. Use the Runge-Kutta method to calculate approximate values of the solution $y = \phi(t)$ of the initial value problem

$$y' = 1 - t + 4y, \quad y(0) = 1.$$

Taking $h = 0.2$, $y(0.2) = ?$

參考用

注意：背面有試題

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(10分) 8. Disscuss the solutions of the system

$$\frac{dx}{dt} = x(1-0.5y)$$

$$\frac{dy}{dt} = y(-0.75+0.25x)$$

for x and y positive.

You need to describe the type of each critical point and draw the phase portrait of this system.

(10分) 9. Consider the heat conduction problem

$$u_{xx} = u_t, \qquad 0<x<30, \quad t>0,$$

$$u(0,t)=20, \qquad u(30,t)=50, \qquad t>0.$$

$$u(x,0)=60-2x, \qquad 0<x<30.$$

Find the steady-state temparature distribution and the boundary value problem that determine the transient distribution.

(10分) 10. Determine the normalized eigenfunction of the problem

$$y''+\lambda y=0, \quad y(0)=0, \quad y'(1)+y(1)=0.$$