# 國立中央大學八十六學年度轉學生入學試題卷

數學系　三年級　　　科目：　微分方程　　　共 1 頁 第 1 頁

總共 9 題，① 至 ⑧ 每題 12 分，⑨ 佔 4分，總分100分。

① Solve for $x$ : $\begin{cases} x'(t) = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} x + \begin{pmatrix} e^{2t} \\ 1 \end{pmatrix} \\ x(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{cases}$.

② Define $e^{At}$ and prove its existence, where $t \geq 0$ and $A = (a_{ij})_{n\times n}$ is an $n\times n$ constant matrix.

③ Show that the Laplace transform $\mathcal{L}\{\sin(\alpha t)\}$ of $\sin(\alpha t)$ exists for each $\alpha \in \mathbb{R}$.

④ Find a particular solution to $y'' + y = f(x)$, where $f(x)$ is continuous.

⑤ Find a general solution to $y''' - 3y'' + 4y = xe^{2x}$.

⑥ Let $f(x)$ be a particular solution to $y'' + p(x)y' + q(x)y = g(x)$ and let $\{y_1, y_2\}$ be a fundamental solution set. Here $p$, $q$, and $g$ are continuous. Show that every solution to $y'' + py' + qy = g$ is of the form $c_1y_1 + c_2y_2 + f(x)$, where $c_1$ and $c_2$ are two constants.

⑦ Let $p$, $q$ be two continuous functions on $(a,b)$ and let $x_0 \in (a,b)$. Show that $\begin{cases} y' + p(x)y = q(x) \text{ on } (a,b) \\ y(x_0) = y_0 \end{cases}$ has a unique solution.

⑧ Let $p$, $q$ be two continuous functions on $(a,b)$ and let $x_0 \in (a,b)$. Let $y_1$ and $y_2$ be two solutions to $y'' + py' + qy = 0$ on $(a,b)$. Show that if $(y_1y_2' - y_2y_1')(x_0) \neq 0$, then every solution $\phi$ to $y'' + py' + qy = 0$ on $(a,b)$ has the form $\phi = c_1y_1 + c_2y_2$ for some constants $c_1$ and $c_2$.

⑨ Solve for $y$ : $xy + y^2 + x^2 - x^2\frac{dy}{dx} = 0 \quad (x > 0)$.