

國立中央大學八十五學年度轉學生入學試題卷

數學系 三年級

科目: 微分方程

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參考用

每題 10 分, 總共 10 題, 總分 100 分

- ① Given that $y = x^{-1}$ is a solution of $2x^2y'' + 3xy' - y = 0$, $x > 0$, find a second linearly independent solution.
- ② Find a particular solution of $y'' - 3y' - 4y = 3e^{2x} + 2\sin x - 8e^x \cos 2x$.
- ③ Find the solution of the initial value problem $y'' - y' + 0.25y = 0$, $y(0) = 2$, $y'(0) = \frac{1}{3}$.
- ④ Solve the differential equation $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$.
- ⑤ Find a particular solution of $y'' + 4y = 3 \operatorname{csc} x$.
- ⑥ Find a series solution in power of x of the equation $y'' - xy = 0$, $-\infty < x < \infty$.
- ⑦ Find the general solution of $x' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} x$, where x is of the form $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.
- ⑧ Show that $\int_0^{\infty} \frac{e^{-st}}{x} \sin t \, dt$ exists for $s > 0$.
- ⑨ Show that the series $(1 + \sum_{n=1}^{\infty} \frac{t^n}{n!})$ converges for each $t \in \mathbb{R}$.
- ⑩ Let y_1 and y_2 be two solutions of the differential equation $y'' + p(x)y' + q(x)y = 0$, where p and q are continuous on an open interval I . Show that the Wronskian $w(y_1, y_2)(x)$ is either zero for all x in I or else never zero in I .