國立中央大學103學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組(一般生)　科目：微積分　共 乙 頁　第 1 頁

本科考試禁用計算器　　　　*請在試卷答案卷（卡）內作答

1. Given an equation

$$P_t = Xe^{-r\tau}N(-d_2) - S_tN(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r + \frac{\sigma_s^2}{2}\right)\tau}{\sigma_s\sqrt{\tau}},\quad d_2 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - \frac{\sigma_s^2}{2}\right)\tau}{\sigma_s\sqrt{\tau}} = d_1 - \sigma_s\sqrt{\tau},$$

$$\tau = T - t, \text{ and } N(d_1) = \int_{-\infty}^{d_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du.$$

a. (15%) It can be easily derived that $\dfrac{\partial N(d_1)}{\partial d_1} = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$.

Please show $\dfrac{\partial N(d_2)}{\partial d_2} = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \cdot \dfrac{S_t}{X} \cdot e^{r\tau}$.

b. (15%) Please calculate $\dfrac{\partial P_t}{\partial S_t}$.

c. (20%) Please calculate $-\dfrac{\partial P_t}{\partial \tau}$.

2. Consider the problem

$$\underset{w_1,w_2,w_3}{\text{minimize}}\ \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{12} + 2w_1w_3\sigma_{13} + 2w_2w_3\sigma_{23}$$

subject to

$$w_1 + w_2 + w_3 = 1$$

$$w_1R_1 + w_2R_1 + w_2R_1 = E^*$$

a. (10%) Please write the Lagrangian.

b. (10%) Please show the first order conditions.

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3. (10%) If $w_1 + w_2 = 1$, please obtain the w_1 and w_2 to minimize the following equation.

$$\sigma_A^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

4. (10%) $U(W) = \begin{cases} \dfrac{W^{1-r}-1}{1-r}, & \text{if } \gamma \neq 1 \\ \ln(W), & \text{if } \gamma = 1 \end{cases}$

Please calculate $R(W) = -\dfrac{U''(W)}{U'(W)}$,

5. (10%) $F(W) = \dfrac{1-\gamma}{\gamma}\left(\dfrac{\alpha W}{1-\gamma} + \beta\right)^{\gamma}$

If $r \neq 1$, $\alpha > 0$, $\left(\dfrac{\alpha W}{1-\gamma} + \beta\right) > 0$, and $\beta = 1$ if $\gamma \to -\infty$, please calculate $-W\dfrac{F''(W)}{F'(W)}$.

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