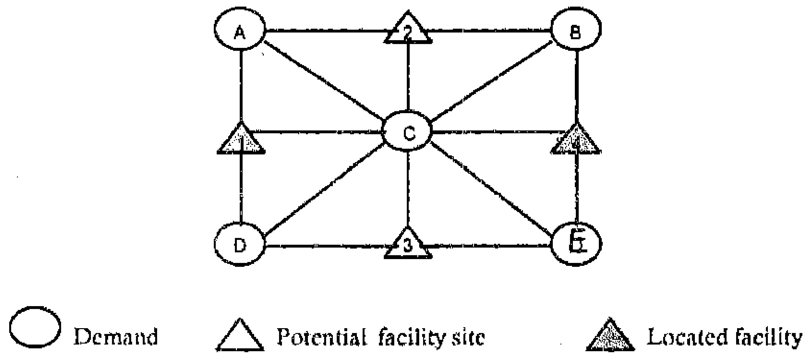
國立中央大學104學年度碩士班考試入學試題

所別：工業管理研究所碩士班 不分組(一般生)　科目：作業研究　共 2 頁 第 1 頁
本科考試禁用計算器　＊請在答案卷（卡）內作答

參考用

1. (30 分) "*The infinite-capacity, fixed-charge location problem*" is concerned with assigning manufacturing facilities to appropriate sites in order to satisfy all customer demands, and the total cost for assigning facilities to sites and transporting products to satisfy customer demand must be as low as possible. The following graph is an example showing 5 demand nodes (A, B, C, D and E; all demand nodes are represented by an oval) and 4 facility sites (1, 2, 3 and 4; all facility sites are represented by a triangle), with facility sites 1 and 4 actually used to make products. Once used, each facility site is assumed to have unlimited capacity that it can make as many products as needed. However, a facility site can satisfy a demand node's demand only if the facility site is connected to the demand node (for example, facility site 2 can satisfy the demand at demand nodes A, B and C, but not the demand at D and E).



Please use the parameters and decision variables defined below to formulate a linear mixed-integer programming model for the infinite-capacity, fixed-charge location problem. Note: Your model must be formulated for the general case, NOT just for the example showing in the above graph. Also note: Your model must have an objective function and one or more constraints, and you are NOT allowed to define any new decision variables or parameters in your model.

Parameters:

- f_j … fixed cost for assigning facilities to facility site j.
- h_i … demand quantity at demand node i.
- d_{ij} … distance from demand node i to facility site j (d_{ij} is a very large number if demand node i is not connected to facility site j).
- c … cost for transporting each unit demand each unit distance.

Decision variables:

- X_j … binary variable such that $X_j = 1$ if facility site j is used, and 0 if otherwise.
- Y_{ij} … fraction of demand at demand node i which will be satisfied by facility site j.

2. (20 分) Please use the simplex tableau method to find an optimal solution to the following problem.

$$\begin{aligned}\text{Minimize}\quad & z = x_1 + x_2\\ \text{Subject to}\quad & 2x_1 + x_2 + x_3 = 4\\ & x_1 + x_2 + 2x_3 = 2\\ & x_1, x_2, x_3 \geq 0\end{aligned}$$

注意：背面有試題

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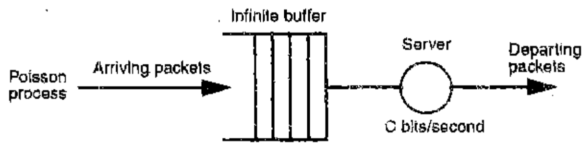
3. (5 * 3 = 15 points)

A switch has an infinite buffer and an infinite number of users generating messages according to a Poisson process with average inter-arrival time of 800 milliseconds. The switch serves requests with a service time that is exponentially distributed with an average service time of 500 milliseconds.



(1) What is the queuing model with the Kendal's notation?

(2) What is the utilization rate the switch?

(3) If the switch is upgraded to reduce average service time to 400 milliseconds. What is percentage of reduction in waiting time?

$$W = \frac{1}{\mu - \lambda} \qquad \rho = \frac{\lambda}{\mu}$$

4. (10 * 2 = 20 points)

A machine deteriorates rapidly and must be inspected and maintained periodically. Inspection declares the machine to be in four possible states:

State 0: Good as new

State 1: Operable, minor deterioration

State 2: Operable, major deterioration

State 3: Inoperable, thus must be repaired

The transition probabilities are

States	0	1	2	3
0	0	7/8	1/16	1/16
1	0	3/4	1/8	1/8
2	0	0	1/2	1/2
3	1	0	0	0

There are costs as system evolves:

State 0: cost $ 0　　State 1: cost $1000　　State 2: cost $3000　　State 3: cost $6000

(1) What are the steady state probabilities?

(2) What is the average cost of this maintenance policy?

5. (15 points)

According to a survey:

11% of those in the lowest income quartile were college graduates.

19% of those in the second-lowest income quartile were college graduates.

31% of those in the third-lowest income quartile were college graduates.

53% of those in the highest income quartile were college graduates.

Each quartile contains 25% of the total observations. What is the probability that a college graduate falls in the lowest income quartile?

$$P(Q_1|G) = \frac{P(G|Q_1)P(Q_1)}{\sum_{j=1}^{4} P(G|Q_j)P(Q_j)}$$

注意：背面有試題