# 國立中央大學105學年度碩士班考試入學試題

所別： 太空科學研究所 碩士班 不分組(一般生)
太空科學研究所 碩士班 不分組(在職生)



科目： 應用數學

本科考試禁用計算器

＊請在答案卷(卡)內作答

請注意：作答時，請寫出推導計算步驟或用文字說明如何獲得答案。如果只列出最後答案，卻沒有推導計算步驟或文字說明，該題將不予計分。

**(1, 20%)** Find a general solution of $y'+y=-\frac{x}{y}$.

**(2, 20%)** Find the solution of the Euler-Cauchy equation. Show the details of your work.

$x^2y''+xy'+9y=0,\ y(1)=0,\ y'(1)=2.5$

**(3, 20%)** Please derive a general solution of the Legendre's differential equation, $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$, in powers of $x$ with convergence range $|x|<1$.

**(4, 20%)** Please prove the following statements: If two functions $f(t)$ and $g(t)$ has the Laplace transform $F(s)$ and $G(s)$, respectively, the product $H(s)=F(s)G(s)$ is the Laplace transform of $h(t)$ given by the following equation.

$h(t)=(f*g)(t)=\int_0^t f(\tau)g(t-\tau)d\tau$

It is noted that the Laplace transform of $f(t)$ is defined as $F(s)=\int_0^\infty e^{-st}f(t)dt$.

**(5, 20%)** Verify that for any $n\times n$ matrices $\mathbf{A}$ and $\mathbf{B}$, $\det(\mathbf{AB})=\det(\mathbf{A})\det(\mathbf{B})$.