

國立中央大學 105 學年度碩士班考試入學試題

所別： 統計研究所碩士班 不分組(一般生)

共 1 頁 第 1 頁

統計研究所碩士班 不分組(在職生)

科目： 基礎數學

本科考試可使用計算器，廠牌、功能不拘

\*請在答案卷(卡)內作答

1. (5%) Find the limit of the sequence  $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$ .

2. (5%) Determine if the following series is convergent,

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2+n+1}$$

3. (20%) Let  $f(x) = 3x^4 - 4x^3 + 1$ .

(a) (5%) Find all critical points of  $f(x)$ .

(b) (5%) Determine if  $f(x)$  has local minimum or local maximum at each critical point in (a).

(c) (5%) Sketch the function graph of  $f(x)$ .

(d) (5%) Find the global minima and global maxima of  $f(x)$  if they exist.

4. (10%) Compute

$$\int_0^2 \int_0^{\sqrt{1-x^2}/4} 1 dy dx.$$

5. (10%) Compute

$$\int_0^{\infty} e^{-x^2} dx.$$

注意：背面有試題

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所別：統計研究所碩士班 不分組(一般生)  
統計研究所碩士班 不分組(在職生)

共 2 頁 第 2 頁

科目：基礎數學

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6. (25%) Let  $f'(x) = df(x)/dx$  be the differentiation of  $f(x)$  with respect to  $x$ . Let  $V$  be the set of real polynomials with degrees less than or equal to three. Suppose that  $T(f(x)) = f'(x)$  is a map from  $V$  to  $V$ .
- (a) (5%) Show that  $T$  is a linear transform. (Recall that a map  $T: V \rightarrow V$  is a linear transform, if for all  $f(x), g(x) \in V$  and some real number  $\alpha$ , then  $T(\alpha f(x) + g(x)) = \alpha T(f(x)) + T(g(x))$ .)
- (b) (5%) Find the kernel of  $T$ .
- (c) (5%) Determine if  $T$  is invertible or not.
- (d) (10%) Let  $\alpha = \{1, x, x^2, x^3\}$  be a basis of  $V$ . Find the matrix of  $T$  with respect to the basis  $\alpha$ .
7. (25%) Let  $A$  be a  $5 \times 5$  matrix,

$$A = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

- (a) (5%) Compute the determinant of  $A$ .
- (b) (10%) Find the characteristic polynomial of  $A$ .
- (c) (10%) Find the Jordan form of  $A$ .

注意：背面有試題