

國立中央大學 105 學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

共 2 頁 第 1 頁

科目： 工程數學(機率、線性代數)

本科考試禁用計算器

*請在答案卷(卡)內作答

1. (15%) The duration (in minutes) of a cell-phone call is modeled as an exponential probability density function. The expected value of the duration of a mobile phone call is 5 minutes.
 - (1) (5%) Find the cumulative distribution function of the duration (in minutes) of a cell-phone call.
 - (2) (10%) Find the standard deviation of the duration (in minutes) of a cell-phone call.
2. (20%) For continuous random variables X and Y with the joint probability density function
 - (1) (10%) Find the expected value of X .
 - (2) (10%) Find the conditional expected value $E[X|Y=0.5]$.
3. (15%) The expected value vector of the continuous random vector $\mathbf{X}=[X_1 \ X_2]^T$

is $[0 \ 0]^T$. The covariance matrix of \mathbf{X} is $\begin{bmatrix} 4 & 3 \\ 3 & 9 \end{bmatrix}$. The continuous random vector

\mathbf{Y} is $[Y_1 \ Y_2]^T$ where $Y_1 = 2X_1 - X_2$ and $Y_2 = X_1 + 3X_2$.

- (1) (5%) Find the covariance matrix of \mathbf{Y} .
- (2) (10%) Find the correlation matrix of \mathbf{Y} .

注意：背面有試題

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4. (5 %) Show that an $n \times n$ matrix A is nonsingular if and only if the column vectors of A form a basis for \mathbb{R}^n .
5. (15 %) Let $T: V \rightarrow W$ be a linear transformation. $\dim(\cdot)$, $\text{Im}(\cdot)$ and $\text{Ker}(\cdot)$ mean the dimension, image and kernel of a subspace (\cdot) respectively.
- (a) (5 %) Show that T is one-to-one if and only if $\text{Ker}(T) = \{0\}$.
- (b) (5 %) Show that if $\dim(V) = \dim(W)$ is finite, then T is one-to-one if and only if T is onto.
- (c) (5 %) Let $T: M_{n \times n} \rightarrow M_{n \times n}$ by $T(A) = CA$ for all A in $M_{n \times n}$.
If $BC = I$ where B and C are $n \times n$ matrices, show that also $CB = I$.
6. (5 %) Let $A = \begin{bmatrix} 4 & 7 & -2 \\ 0 & 25 & 4 \\ 0 & 0 & 9 \end{bmatrix}$. Find a matrix B such that $B^2 = A$.
7. (10 %) Give that an invertible matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ can map the line $y = 5x + 2$ into another line y' . Find the equation of the line y' .
8. (15 %) Use the eigenvalues method to solve the initial value problem of a system with linear differential equations:
- $$y_1'' = -3y_2 + y_1' + 3y_2'$$
- $$y_2'' = 3y_1 + 3y_1' - y_2'$$
- $$y_1(0) = 1, y_2(0) = 0, y_1'(0) = -3, y_2'(0) = 2$$
- (a) (2 %) Give the coefficient matrix for this system.
- (b) (4 %) Find the eigenvalues of this system.
- (c) (4 %) Find the eigenvectors corresponding to all eigenvalues.
- (d) (5 %) Find the solution of this system.

注意：背面有試題