

國立中央大學 105 學年度碩士班考試入學試題

所別： 數學系碩士班 應用數學組(一般生)

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數學系碩士班 應用數學組(在職生)

科目： 微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

- 1.(10%) Prove the series $\sum_{n=1}^{\infty} n2^{-n} \sin \frac{1}{n}$ is convergent.
- 2.(10%) Find the limit $\lim_{x \rightarrow 0} \left(\frac{2}{x \sin^{-1} x} - \frac{1}{x^2} \right)$.
- 3.(10%) Find the interval of convergence and sum of the power series $\sum_{n=1}^{\infty} \frac{x^{4n-1}}{4n-1}$.
- 4.(10%) Compute the integral $\int \frac{2}{2 + \cosh x} dx$.
- 5.(10%) Let $f(x) = \int_0^{x^2} \tan^{-1} \left(\frac{t}{x^2} \right) dt$. Find the derivative of $f(x)$.
- 6.(10%) Find the point which is closest to the origin on the curve of intersection of the plane $2y + 4z = 7$ and the cone $z^2 = 4x^2 + 4y^2$.
- 7.(10%) Compute the double integral $\int \int_D \frac{xy^2}{1+x^2y^2} dA$, here D is the region bounded by the curves $xy = 1$, $xy = 4$, $x = 1$ and $x = 4$.
- 8.(10%) Compute the triple integral $\int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-x^2-2y^2-\frac{1}{3}z^2} dx dy dz$.
- 9.(10%) Find the volume of the solid which is bounded by the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $r = 4 \cos \theta$.
- 10.(10%) Compute the line integral $\int_C (e^x \sin y + 5y) dx + (e^x \cos y + 3x - 4y) dy$ counter-clockwise around the ellipse $C : 4x^2 + 9y^2 = 36$.