1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $y' + xy = x/y$ with the initial condition of $y(0) = 3$　　(5%)

(b) Find the solution for the general solution for the ODE $y'' + 4y' + 4y = e^{-x}\cos x$　　(5%)

(c) Solve the nonhomogeneous linear ODE $x^2 y'' - xy' - 3y = x^2$ by variation of parameters or undetermined coefficients.　　(5%)

(d) Solve the ODE $\left(D^3 - 3D^2 + 3D - I\right)y = 4\cos x$.　　(5%)

(e) $\begin{cases} y_1' = y_1 + y_2 + 10\cos x \\ y_2' = 3y_1 - y_2 - 10\sin x \end{cases}$, find y_1 and $y_2 = ?$　　(5%)

2.

Given an ODE: $y'' + 2y' - 3y = u(t-1) * \dfrac{1}{t^2+1}\delta(t-1)$, where $u(t)$ is unit step function, $\delta(t)$ is impulse function, symbol $*$ denotes convolution, $L(y) = Y(s) = \int_0^\infty y(t)e^{-st}dt$, $y(0) = y'(0) = 0$.

(a) Calculate $L[u(t-1) * \dfrac{1}{2}\delta(t-1)] = ?$　　(10%)

(b) Find the solution $y(t) = L^{-1}[Y(s)]$ of the above ODE.　　(15%)

3.

(1) Evaluate the surface integral $\displaystyle\int_A (x\hat{i} + y\hat{j} + z\hat{k})\cdot\hat{n}\,dA$ over a unit cube. The cube has the volume center located at the origin (0, 0, 0), and its six plane surfaces are perpendicular to the x-, y-, and z- axes respectively, and $\hat{n}$ is the unit normal vector pointing outwards from the surface.　　(5%)

(2) Evaluate the contour integral $\displaystyle\oint x\,dy - y\,dx$ around the following closed curves in the x-y plane.

(a) The curve comprises four sides of a unit square whose center is located at the origin (0, 0). (5%)

(b) The curve is the circumference of a unit circle whose center is located at the origin (0, 0). (5%)

(3) Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A.　　(5%)

(b) Find the eigenvalues and eigenvectors of A^5.　　(5%)

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4.

(a) An arithmetic series is defined by the following:

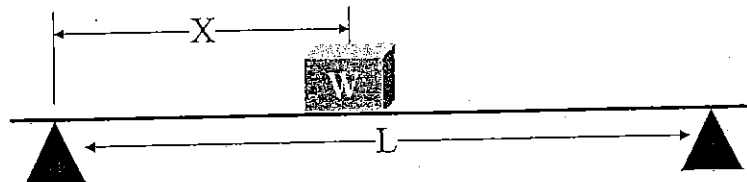$$a + (a + d) + (a + 2d) + (a + 3d) + \dots + [(a + (n - 1)d]$$

　　a is the first term.

　　d is the "common difference."

　　n is the number of terms to be added.

Using this formula, write a **function** that accepts the value of a, d and n as parameters from a calling function. The return value of this function should be the sum of the arithmetic series. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. The variables are declared to real numbers except for n as integer. **(10%)**

(b) Write a program to calculate and display the maximum bending moment, M, of a beam that's supported on both ends (see Figure). The formula is $M = XW (L - X) / L$, where X is the distance from the end of the beam that a weight, W, is placed, and L is the length of the beam. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. All variables are declared to real numbers. **(15%)**

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