國立中央大學 105 學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 熱流組(一般生)　　共 2 頁 第 1 頁

機械工程學系光機電工程 碩士班 光機組(一般生)

能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $y'+xy = x/y$ with the initial condition of $y(0)= 3$ (5%)

(b) Find the solution for the general solution for the ODE $y'' + 4y' + 4y = e^{-x}\cos x$ (5%)

(c) Solve the nonhomogeneous linear ODE $x^2y'' - xy' - 3y = x^2$ by variation of parameters or undetermined coefficients. (5%)

(d) Solve the ODE $\left(D^3 - 3D^2 + 3D - I\right)y = 4\cos x$. (5%)

(e) $\begin{cases} y_1' = y_1 + y_2 + 10\cos x \\ y_2' = 3y_1 - y_2 - 10\sin x \end{cases}$, find y_1 and y_2 =? (5%)

2.

Given an ODE: $y'' + 2y' - 3y = u(t-1) * \frac{1}{t^2+1}\delta(t-1)$, where $u(t)$ is unit step function, $\delta(t)$ is impulse function, symbol $*$ denotes convolution, $L(y) = Y(s) = \int_0^\infty y(t)e^{-st}dt$, $y(0) = y'(0) = 0$.

(a) Calculate $L[u(t-1) * \frac{1}{2}\delta(t-1)] = ?$ (10%)

(b) Find the solution $y(t) = L^{-1}[Y(s)]$ of the above ODE. (15%)

3.

(1)Evaluate the surface integral $\int_A (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{n} dA$ over a unit cube. The cube has the volume center located at the origin (0, 0, 0), and its six plane surfaces are perpendicular to the x-, y-, and z- axes respectively, and $\hat{n}$ is the unit normal vector pointing outwards from the surface. (5%)

(2)Evaluate the contour integral $\oint x dy - y dx$ around the following closed curves in the x-y plane.

(a) The curve comprises four sides of a unit square whose center is located at the origin (0, 0). (5%)

(b) The curve is the circumference of a unit circle whose center is located at the origin (0, 0). (5%)

(3) Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A. (5%)

(b) Find the eigenvalues and eigenvectors of A^5. (5%)

注意：背面有試題

國立中央大學105學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 熱流組(一般生)　　
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

4. Partial differential equations (PDEs)

(a) Use separation of variables to find the solution of $y\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=0$ **(10%)**

(b) Solve the boundary value-problem $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-h$, where h is a positive constant, $h>0$. The boundary conditions are given as: $\begin{cases} u(0,y)=0,\ u(\pi,y)=1,\ y>0 \\ u(x,y)=0 \text{ as } y\to\infty \\ u(x,0)=0,\ 0<x<\pi \end{cases}$ **(15%)**

注意：背面有試題