科目<u>微積分</u>類組別<u>A3 A4 A6</u> 共<u>/</u>頁 第<u>/</u>頁 *請在答案卷內作答

1. (10 points): Let

$$I = \int_{C} \frac{y}{x^2 + y^2} dx - \frac{x}{x^2 + y^2} dy$$

where C is a circle oriented counterclockwise.

- (a) Evaluate I if C is given by $(x-2016)^2 + (y-2016)^2 = 1$.
- (b) Evaluate I if C is given by $x^2 + y^2 = 1$.
- **2.** (10 points): Find the maximum and minimum values of the function $f(x, y, z) = x^2 y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$.
- 3. (10 points): Compute

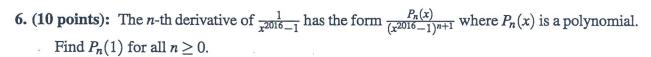
$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$$

4. (10 points): For what positive x does the following series converge?

$$\sum_{n=1}^{\infty} \left(\sqrt[n]{x} - 1 \right)$$

5. (10 points): Let $B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \le 1\}$. Evaluate the integral

$$\iiint\limits_{R} \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4} \, dV.$$



7. (20 points): (a) Prove that

$$\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx = \int_0^\infty \frac{\sin x}{x} dx.$$

(b) Evaluate the improper integral

$$\int_0^\infty \frac{\sin x}{x} \, dx.$$

- **8.** (10 points): For each continuous function $f:[0,1] \to \mathbb{R}$, let $I(f) = \int_0^1 x f(x)(x-f(x)) dx$. Find the maximum value of I(f) over all such functions f.
- 9. (10 points): Evaluate

$$\int_0^\infty \left(x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots \right) \left(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \right) dx.$$