

國立中央大學 106 學年度碩士班考試入學試題

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科目：統計

本科考試禁用計算器

*請在答案卷 內作答

參考用

問答題

答題請依照題目順序、標示題號、簡捷作答，並皆須提示理由解釋、計算過程或證明，否則不予計分。

1. 已知 $\{X_1, X_2, \dots, X_n\}$ 為一組樣本數為 n 從常態 $N(3, 4)$ 所抽出的 i.i.d. 隨機樣本。試回答下列是非題。(不論你認為題意正確、錯誤或不一定皆須提示理由解釋，否則不計分。)(30%)

1.1 Law of Large Number suggests that $\frac{\sum_{i=1}^n X_i^2}{n}$ tends to be 3^2 when $n \rightarrow \infty$.

1.2 Central limit theorem suggests that any random variable can be properly standardized to follow or apply Normal distribution.

1.3 If $s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$ is the sample variance, $\frac{(n-1)s^2}{4} \sim \chi^2(n)$.

1.4 Let $Y_i \equiv \frac{X_i - 3}{2}$, $i = 1, 2, \dots, n$, and $Q \equiv (\sqrt{3}Y_1 + \sqrt{5}Y_2 + Y_3 + \sqrt{2}Y_4)^2 + (Y_5 + \sqrt{2}Y_6 - 2Y_7)^2$, then Q is a χ^2 distributions with degree of freedom as 7.

1.5 If (X_1, X_2) are a random sample from a Bernoulli distribution, which takes the value of 1 with probability p . Suppose you are asked to test $H_0 : p = 1/2$ against the alternative hypothesis $H_a : p \neq 1/2$, and you decide to reject the null hypothesis whenever $|\frac{1}{2}(X_1 + X_2) - \frac{1}{2}| \geq 1/8$, then the size (prob. of type I error) of the test is $1/4$?

1.6 假設某“國運昌隆黨”想推估國民中有多少的人口比例 p 支持 DPP 黨推出的“無核家園”立法，試算在 95% 的信心水準下，如果欲控制誤差範圍不超過千分之一，則至少需要抽樣樣本數 1000 才夠。

2. 假設某協會主張對於酒駕肇事的駕駛處以下的隨機徒刑：由酒駕犯同時投擲一枚公正的銅板與一個公平的骰子，如果銅板正面則吊銷執照、訓斥改過、在上滿時數的道安講座後即開釋；如果銅板是反面，則除了吊銷執照以外，還以拘役伺候，刑期年份長短則由骰子所決定(每點 2 年)。試問在此設計的隨機機制下，一個酒駕肇事犯大約需要吃多少年的牢飯？他被拘役超過七年的機率為多少？(10%)

3. 曉英發現，若以下列的資料生成方式所得的 $\{(X_t, Y_t)\}_{t=1}^n$ 配對組合，

$$X_t = X_{t-1} + \varepsilon_{1,t}, \varepsilon_{1,t} \stackrel{i.i.d.}{\sim} N(0, 1),$$

$$Y_t = Y_{t-1} + \varepsilon_{2,t}, \varepsilon_{2,t} \stackrel{i.i.d.}{\sim} N(0, 1), \varepsilon_{1,t} \text{ 與 } \varepsilon_{2,t} \text{ 相互獨立,}$$

配適簡單線性迴歸方程式 $Y_t = \alpha + \beta X_t + u_t$ ，常常都可以得到顯著的 β 係數 t 值與很高的 R^2 。試以你學過的統計相關知識評論是否曉英哪裡做錯了？何以會有上述的現象？(10%)

注意：背面有試題

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內作答

4. Consider an linear ordinary least-squares (OLS) model between Y and X as

$$Y_i = \alpha + \beta X_i + \varepsilon_i, i=1, \dots, N.$$

Assume the residual terms ε_i are independently and identically distributed with zero mean and variance σ^2 , that is, ε_i are $iid(0, \sigma^2)$.

- (a) Write down the Normal Equations. (6%)
- (b) Derive the least squares (LS) estimators of α and β . (6%)
- (c) Derive the variance of the LS estimators of α . (6%)
- (d) Suppose now the residuals have mean unequal to zero, $E(\varepsilon_i) = c_i \neq 0$. Is the LS estimator of β unbiased? (6%)
- (e) Suppose now we have an estimator $\bar{\beta} = \sum_{i=1}^N X_i Y_i / \sum_{i=1}^N X_i^2$. Is $\bar{\beta}$ unbiased? (6%)
5. The p.d.f. a random variable x is $f(x) = 1/c$, where $0 < x < c$. We want to test the null hypothesis $H_0: c = 4/3$ versus the alternative hypothesis $H_1: c = 7/3$. Suppose we perform the test in the following manner. We draw an observation of x , and reject H_0 when the drawn $x > 1$.
- (a) What is probability of committing Type I error? (5%)
- (b) What is probability of committing Type II error? (5%)
6. Suppose a given stock's weekly returns $\{r_i\}$ is distributed with mean 0 and variance σ^2 , and there exists equal covariance across $\{r_i\}$; that is, $Cov(r_i, r_j) = c$ for all i and j . Define three-week return $r_{t,t+2} = r_t + r_{t+1} + r_{t+2}$, and define variance ratio

$$VR = \frac{Var(r_{t,t+2})}{3Var(r_t)}$$

Show $VR = 1 + 2\rho$, where ρ is the correlation between r_t and r_{t+1} . (5%)

7. Corporate governance researchers typically believe that firm value would be hurt if a CEO also holds the position of the chairman of the board of directors (such situation is denoted as CEO-chairman duality). In order to find empirical support to this belief, we gather a sample of 10,000 firms around the world. Also suppose we measure firm value with Tobin's Q, computed as the market value of the firm divided by the firm's total assets. Please describe how we perform the test. Here we do not consider any other factors that may affect firm value. *Be sure to write down the null and alternative hypotheses, the test statistic, and the decision rule.* (5%)